Product Bundling and Incentives for Merger and Strategic Alliance

Sue H. Mialon*

June 2009 (This Version: July 2011)

Abstract

This paper analyzes firms' choice between a merger and a strategic alliance in bundling their product with other complementary product. We consider a framework in which firms can improve profits only from product bundling. While mixed bundling is not profitable, pure bundling is because pure bundling reduces consumers' choices, and thus, softens competition among firms. Firms benefit the most from this reduced competition if they form an alliance. Firms do not gain as much from a merger because, internalizing the complementarity between the two products, a merged firm is inclined to pursue aggressive pricing to gain market share. Yet, firms may be motivated to choose a merger over an alliance because of foreclosure possibility as foreclosure is not possible under strategic alliance. However, in response, unmerged rivals can use a strategic alliance to avert foreclosure. Hence, the possibility of counter-bundling via strategic alliance by rivals reduces the incentives for merger. In equilibrium, bundling is offered only through strategic alliances.

JEL Classification Numbers: L13, L41, L11, D21, D43, L21.

1 Introduction

This paper models firms' strategic choice between merger and strategic alliance when the only way to improve their profits is through bundling their product with other complementary product. In 2001, the European Commission (EC) blocked the $42 billion merger between GE and Honeywell, which had been approved earlier by US antitrust authorities. The primary reason to block the merger was that it would facilitate the bundling of aircraft engines and systems, which would make it difficult for competitors to mimic, and thus, such a bundling would lower welfare by eliminating a

*Department of Economics, Emory University, Atlanta, GA 30322 (e-mail: smialon@emory.edu).
For helpful comments, I am extremely grateful to Mark Armstrong, R. Preston McAfee, Kaz Miyagiwa, and Russell Pittman.

sufficient number of competitors from the market (Hewitt, 2002). Many studies were motivated by this case, but several questions remain unanswered: (1) Is it bundling or merger that raises antitrust concerns? If bundling is anticompetitive, why do we need to block a merger instead of bundling? Does merger make bundling anticompetitive? (2) Would firms want to merge to bundle their products? Is bundling most profitable when firms merge? Is merger the only way of offering bundling for firms? (3) If bundling is not accompanied by a merger, would there be no antitrust concerns for bundling?

To answer these questions, we consider a model where firms choose how to bundle their product with other complementary product. Instead of merging, firms could use strategic alliances. For instance, AT&T currently offers a bundle of its phone and Internet components with a satellite TV component through a strategic alliance with DirecTV to counter TV component options in the bundling provided by rival cable companies. Thus, the focus of this paper is on examining when firms choose to merge and when firms choose to form a strategic alliance to offer bundling, which would answer (2).

To answer questions (1) and (3), we consider markets for two products that are unrelated in production. Therefore, a merger between the two complementary good producers does not induce synergy. Firms can improve profits only from bundling their products together. Removing the possibility of efficiency gains from a merger, we can focus on how merger affects the profitability of bundling. As it is also feasible for firms to offer bundling via a strategic alliance, we can show how merger and strategic alliance improve the profitability of bundling differently.

In each market, there are two firms in competition wherein the products are horizontally differentiated in that consumers must incur transportation costs to reach firms. We assume that the transportation cost is higher in the second market than in the first market. Thus, prior to bundling, the firms in the second market enjoy higher market power and higher profits than the firms in the first market. The model consists of multi-stage games. First, firms decide whether to merge with a complementary product producer in order to sell their products in a bundle. If they do not merge, but they still wish to offer bundling, they can do so through a strategic alliance. Firms decide whether to offer mixed bundling, pure bundling, or no bundling. Then, they determine their prices at a later stage. We first analyze the case of a unilateral merger/strategic alliance and a unilateral bundling decision by one pair of firms. We then incorporate the possibility of counter bundling via strategic alliance by unmerged rivals. Last, we consider the full game where all firms choose between merger and alliance in order to offer bundling and we derive the equilibrium merger/strategic alliance decisions.

In the case of a unilateral bundling by a merged firm or allied firms, we find that mixed bundling is not profitable. Allied firms do not gain from mixed bundling due to their free-riding incentives. Each firm wants to free ride on the other firm's bundle discount offer to increase the sales of bundled products. Thus, no discount is offered in the end. Mixed bundling is unprofitable for a merged firm because the merged firm sells a bundle at a discount. To sell a bundle, the firm needs to offer a discount. Otherwise, consumers will simply purchase two stand-alone products as before. Such a discount is intended to bring about a higher market share for the merged firm and would improve the profits if the merged firm can make up for the discount by charging more onto consumers who do not purchase a bundle. However, when the merged firm offers a bundle discount, rivals respond to the discount by cutting their stand-alone prices. Then, the competitive pressure on the stand-alone prices makes the merged firm unable to increase the stand-alone prices much. Thus, the mixed bundling becomes unprofitable.

On the other hand, firms always profit from pure bundling via strategic alliance. When two products are sold only in a bundle, consumers have only two options: either to buy a bundle from one pair of merged or allied firms or to buy the two products from the other firms. In this case, how much cost consumers incur in order to shop around between firms for the first product becomes the same as the cost for the second product because the first product cannot be sold separately without the second product and vice versa. This means that for the first product, firms can charge as high as they can for the second product. Hence, given the complementarity of the two products, removing the possibility of separate sales, pure bundling enhances the firms' market power. Under strategic alliance, allied firms independently determine the price for their own component in a bundle, and thus, the bundle price is the sum of the two independently determined prices. As the two prices are equally high, the bundle price becomes higher than the sum of two pre-bundling prices, resulting in higher profits for allied firms. In the case of merger, by contrast, there is an additional effect that the merger brings into the market. The merged firm also realizes that by internalizing the complementarity between the two products (the Cournot effect), it can price more aggressively to increase its market share. As a result, a merger entails more intense price competition with rivals, which is why the profits are lower than the profits under strategic alliance and in some cases, the merger may not even be profitable. Yet, the strategic advantage of a merger is that if rivals are not merged, at least one of the rival firms incurs losses from the enhanced price competition. Thus, in some range of parameters, the merger is profitable and reduces the rival's profits enough to force the rival to withdraw from the market. In this case, firms choose to merge with an intention to foreclose competition. However, this possibility of profitable merger and exclusionary bundling exists only if the merger is unilateral.

Moreover, when we introduce the possibility of counter-bundling via strategic alliance by unmerged rivals, we find that rivals can avert foreclosure via strategic alliance. Though the merge-and-bundle strategy incurs losses only to the rival in the second market, foreclosure occurs in both markets. If the rival in the second market does not exit, the rival in the first market would enjoy a profit increase after the merger since pure bundling softens competition. However, as the rival in the second market exits, the rival in the first market alone cannot survive since there is no demand for its stand-alone product. Knowing this, the rival in the first market may be willing to share its profits with the other rival firm to keep the firm alive. As long as the losses of the rival in the second market are smaller than the profit increase for the rival in the first market, rivals will be able to avert foreclosure with profit sharing under a strategic alliance. As a result, under the possibility of strategic alliance, merger is less likely to occur given that the possibility of foreclosure is the only motive for merger. Counter-bundling does not improve the profitability of the unmerged rivals. The only purpose of the counter-bundling via strategic alliance in this case is to enforce the profit-sharing rule that prevents foreclosure.

An example of a strategic alliance for bundling that aims to counter bundling by a merged firm (multi-product firm) is observed in the markets for game consoles and disc formats. By 2004, the battle had deepened between Blu-ray, led by Sony, and HD-DVD, led by Toshiba, over the market for the format of the next generation blue laser technology high-definition DVDs. Independently, Microsoft's Xbox had been competing against Sony's Playstation in the market for game consoles. In September 2006, Sony announced Playstation 3 would use Blu-ray disc drive. In response, Toshiba and Microsoft entered into a partnership that allowed the Xbox 360 consoles to support HD DVD format. This alliance ended in 2008 when Toshiba announced that it would no longer develop, make or market HD DVD players and recorders.

In the full game where all firms simultaneously determine whether to merge and what type of bundling to offer, we find that mixed bundling is a weakly dominated strategy in every subgame. In equilibrium, in a wide range of parameters, firms do not merge but build alliances to offer pure bundling. All firms profit from the pure bundling, but the profit increase is at the expense of consumer welfare. On the other hand, there exists a parameter range where all firms merge with an intention to foreclose competition, but bundling is never offered since bundling is no longer profitable when all firms are merged.

Our model is related to that of Gans and King (2006), which analyzes the profitability of mixed bundling under strategic alliances. In their model, allied firms first decide whether to agree on offering a fixed discount for consumers purchasing bundled products. Once decisions on discounts are made, firms then independently choose their prices. This sequential structure of pricing decisions, which represents firms' commitment to reward consumer loyalty, makes it possible for firms to internalize the impact of bundled discounts, if any, on the prices of stand-alone products. As a result, bundled discount inflates the stand-alone prices of allied firms, which generates the profit from bundling. But this "co-branding," is profitable only if it is unilateral. If both pairs of firms offer co-branding, no firms gain. Gans and King (2006) also consider how the profitability of bundled discounts changes when the firms are integrated. They find that when only one pair of firms are merged, only the merged firm offers a discount, and compared to the case prior to merger, the discount is lower. However, if both pairs of firms merge, no firm offers a bundle discount. In equilibrium, both pairs of firms merge and no discount is offered.

In the present paper, we adopt a different structure of strategic alliance simply because the type of strategic alliance in Gans and King (2006) does not emerge in equilibrium. In this paper, a bundle discount and stand-alone prices are simultaneously determined. Each of the allied firms independently quote for a price (with or without a discount) for their own component in a bundle, and then, the bundle price becomes the sum of the two prices. In the case of pure bundling, many technologically tied products resemble this type of strategic alliance. For example, consider markets for game consoles and video games. There are two types of game consoles: Blu ray format and HD DVD format. The two are incompatible. Video games and game consoles are technologically tied in that video games are written either in Blu-ray format or in HD DVD format. Thus, consumers cannot mix and match the products (e.g., buying Playstation 3 with HD DVD games). In this case, all firms independently set their prices. But the consumption always occurs in a bundle, and the bundle prices are just the sum of the two independently determined prices.[1] While Gans and King (2006) considers mixed bundling only, in this paper, firms choose whether to offer mixed, pure, or no bundling. In this paper, in equilibrium, firms do offer pure bundling via strategic alliance.

Moreover, this paper sheds a light on another strategic purpose of alliance to bundle. When merger facilitates exclusionary bundling, unmerged rivals can use strategic alliance to counter the merger and avert foreclosure. Both this paper and Gans and King (2006) predict that in equilibrium, no bundling is offered via merger since a counter-merger eliminates the profitability of bundling. However, this paper further shows that even if a counter-merger is not readily available for rivals, as it is possible for them to use a strategic alliance to reduce the possibility of foreclosure (which is the only incentive for merger), merger is less likely to occur in equilibrium.

Several papers consider the possibility of using bundling to foreclose competition or to deter entry.[2] Whinston (1990), Choi and Stefanadis (2001), and Nalebuff (2004) analyze how a monopolist can use bundling to foreclose entry. Whinston (1990) shows that a multi-product firm can leverage its monopoly power in one market onto another market with greater competition by "committing" to tying its monopoly product with

[1] In the case of mixed bundling, while it is debatable which type of alliance is more relevant, both the present paper and Gans and King (2006) predict that mixed bundling via strategic alliance in either type may not be easily observed because mixed bundling is unprofitable, it is a dominated strategy, and it never occurs in equilibrium.

[2] On the other hand, many other studies show bundling is likely to increase consumer welfare through lower prices and cost savings. In a general framework of heterogeneous and elastic demands, Armstrong and Vickers (2010) show that mixed bundling leads to welfare gains if consumers need to incur an extra "shopping cost" in order to purchase products from more than one firm and if consumer preferences for product brands are correlated.

Often, bundling is used as a device for price discrimination or product differentiation, in which case the welfare implications are mostly ambiguous. For models on bundling as a device for price discrimination in a monopoly framework, see, for example, Adams and Yellen (1976), McCain (1987), McAfee et al. (1989), DeGraba (1994), and Armstrong (1999). For a model about bundling as a device for product differentiation, see Chen (1997), for example.

a product in the other market. In this way, the monopolist commits to aggressive competition, which makes rival firms in the other market unable to survive.[3] Choi and Stefanadis (2001) consider the entry-deterrence effect of commitment on bundling in the framework of dynamic R&D decisions. In their model, an incumbent deters entry by committing to tying two complementary products as it lowers the R&D investment of entrants and, as a consequence, the probability of entry. Nalebuff (2004) shows that an incumbent with market power in two markets can effectively defend both markets against entry by bundling the two goods, because bundling makes it difficult for a rival with only one of the two products to enter the market.[4]

Economides (1993) and Choi (2008) discuss how merged firms benefit from mixed bundling. Economides (1993) models equilibrium bundling decisions between two multi-product firms. Using the same framework, Choi (2008) examines the effect of a merger on market prices and welfare. Both studies show that unilateral bundling by a merged firm is profitable as the merger internalizes the effect of price competition within the merging partners (the Cournot effect). By contrast, using a different multi-product duopoly model, Matutes and Régibeau (1992) and Armstrong (2006) show that mixed bundling is not profitable for the multi-product firms. It is a dominant strategy for firms not to offer mixed bundling. The model in Matutes and Régibeau (1992) and Armstrong (2006) is equivalent to the case where both pairs of firms merge and offer mixed bundling in this paper.

While Matutes and Régibeau (1992), Economides (1993), Armstrong (2006), and Choi (2008) focus on explaining how bundling affects firms' profits for a given merger, in the present paper, we examine how likely it is that firms actually choose to merge in order to offer bundling. We find that in a wide range of parameters, firms would rather choose strategic alliances to offer product bundling. Firms would not be interested in mergers unless foreclosure is what they are after. Choi (2008) also discusses this possibility that bundling followed by a merger can lead to foreclosure. In Choi (2008), foreclosure is not the only purpose of a merger, and if not combined with bundling, mergers generate a higher consumer surplus. By contrast, in this paper, we model a merger that aims only to induce foreclosure. This type of merger would be more of concern to antitrust authorities. Yet, we find that such a merger is less likely to occur if firms can counter bundling via strategic alliance to avert foreclosure. By modeling firms' choices over merger and strategic alliance, this paper provides more comprehensive understanding about oligopoly bundling and merger motives.

The paper proceeds as follows. Section 2 outlines the benchmark case prior to bundling. In Section 3, we compare two different ways to bundle, namely, bundling

[3] Carbajo et al. (1990) investigates whether a monopolist has strategic incentives to bundle even in the cases in which bundling does not affect the entry or exit decisions of rivals. They show that a firm with monopoly power in one market bundles its product with another product that is sold in competition with rivals because bundling softens competition.

[4] See also *United States v. Microsoft,* 253 F.3d 34, 87 (D.C. Cir. 2001) and Carlton and Waldman (2002) for more discussion on the use of bundling as a method of strategic foreclosure.

through a strategic alliance and bundling through a merger. In Section 4, we characterize equilibrium bundling strategies and show when firms merge and when firms choose strategic alliance to bundle. In Section 5, we discuss an alternative form of strategic alliance and the stability of strategic alliance. Section 6 concludes.

2 Benchmark

The model extends the standard differentiated products model used in Matutes and Régibeau (1992). In this section, we describe the equilibrium prices and profits for firms prior to bundling as a benchmark case. Consider two markets, market 1 and market 2. Consumers purchase at most one unit for each of two complementary products. We assume that consumer valuations for the two goods, v_1 and v_2, are high enough to guarantee that the two markets are fully covered. Moreover, the two goods are complementary in the sense that $v_{12} \gg v_1 + v_2$, where v_{12} is the value of consuming both goods. Yet, consumers differ in their ideal levels of differentiation for the two goods. A continuum of consumers of mass N are uniformly distributed over an interval $[0, 1]$ in each market i, $i = 1, 2$. These characteristics of consumers in the two markets are not correlated, and thus, a consumer's characteristics are given by $x_{12} = (x_1, x_2) \in [0, 1] \times [0, 1]$. In each market i, $i = 1, 2$, there are two firms, A_i and B_i, competing á la Hotelling. Let a_{iA} and a_{iB} be the locations of firms A_i and B_i, respectively. We assume that $a_{iA} = 0, a_{iB} = 1$. Firms are engaged in price competition. Let p_{ij} be the price of firm j in market i, for $i = 1, 2$, and $j = A, B$ prior to bundling. A consumer located at x_i in market i incurs a disutility of $t_i {x_i}^2$ ($t_i(1-x_i)^2$) from purchasing the product from firm A_i (B_i). Without loss of generality, assume that $t_2 \geq t_1 > 0$.Then, in market i, for a given p_{ij}, there is a consumer who is indifferent between the two firms A_i and B_i, and for the consumer,

$$v_i - p_{iA} - t_i(x_i)^2 = v_i - p_{iB} - t_i(1 - x_i)^2.$$

Thus, firms' demands are $D_{iA}(p_{iA}, p_{iB}) = N\{(p_{iB} - p_{iA})/2t_i + 1/2\}$ and $D_{iB}(p_{iA}, p_{iB}) = N\{1/2 - (p_{iB} - p_{iA})/2t_i\}$, for $i = 1, 2$. For simplicity, we assume that the two firms in each market i are symmetric in that they have an identical constant marginal cost of production, i.e., $c_{iA} = c_{iB} = c_i$, for $i = 1, 2$. Each firm must incur a fixed cost of f_i to produce. Then, from the first-order conditions, the profit-maximizing prices are

$$\begin{aligned} p^*_{iA} &= p^*_{iB} = t_i + c_i, \text{ and} && (1) \\ \pi^*_{iA} &= \pi^*_{iB} = \frac{N}{2} t_i - f_i, \text{ for } i = 1, 2. && (2) \end{aligned}$$

We assume that $\frac{N}{2} t_i \geq f_i$.

3 Strategic Alliance v. Merger

Now suppose that firms are deciding whether to merge or to form a strategic alliance with a complementary product producer in order to bundle their products. Since there is no cost synergy from a merger, the merger between two complementary product producers does not alter any market conditions as long as the merger does not affect the transportation cost t_i (the taste parameter) in the two markets. This parameter t_i in market i represents how much consumers value product differentiation for the product i. Thus, it is specific to each product i. For a consumer who buys good 1 from firm A_1, the key factor is how she would feel if she were to buy the product from B_1 instead. It does not matter whether good 1 is now produced by the merged entity between A_1 and A_2 or by A_1 alone. Hence, the merger cannot alter t_i, $i = 1, 2$.[5] For the same reason, bundling the two products does not alter the taste parameters. Bundling alters only the market competition among firms. How merger and strategic alliance enhance or reduce the effect of bundling on market competition will be the focus of interest for firms.

Throughout this paper, the timing of the game among the players is as follows: At stage 0, firms decide whether to merge. At stage 1, firms decide whether to offer mixed, pure, or no bundling. At stage 2, firms set their prices, and at stage 3, consumption occurs.

Firms interact differently depending on whether or not they are merged, whether they offer bundling, and if they do, whether it is mixed bundling or pure bundling. Firms may not want to merge but still desire to offer bundling. In that case, they can offer bundling through a strategic alliance.

We have three cases to consider: a pair of firms merge while the others do not, both pairs of firms merge, and no firms merge but firms offer bundling via strategic alliances. In this section, we first analyze the case when only one pair of firms, A_1 and A_2, make a decision to merge and bundle. We characterize the outcomes when they offer either mixed bundling or pure bundling through merger or strategic alliance. Later in this section, we incorporate the possibility that the unmerged firms B_1 and B_2 can form a strategic alliance to offer bundling. Then, in section 4, we provide a full description of equilibrium merger/strategic alliance decisions by all the firms considering all the possible scenarios.

[5] If the transportation costs are the "actual" cost of travel, the costs depend only on the travel distance but not on the type of the product. That is, $t_1 = t_2 = t$. In this case, mergers do not affect the traveling cost as long as merged firms keep shelving their products independently as before the merger. Alternatively, if the merged firms shelve the two products together in a bundle, the cost of traveling is reduced by half. (See Armstrong [2010] for a similar type of bundling that reduces shopping cost.) However, even in this case, the cost reduction is due only to bundling, not to merger. We interpret t_i as a taste parameter rather than the actual traveling cost because the former represents more general cases.

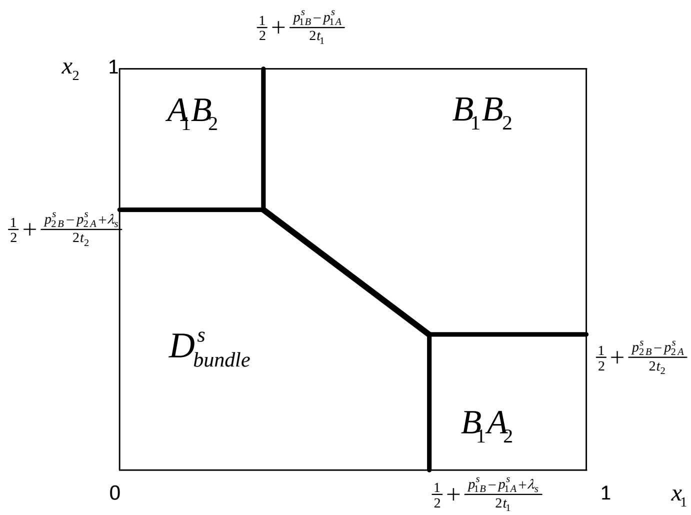

Figure 1: Market demands for a bundle and stand-alone products under mixed bundling

3.1 Unprofitable Mixed Bundling

3.1.1 Strategic Alliance

Suppose A_1 and A_2 form a strategic alliance to offer mixed bundling. Under strategic alliance, firms independently determine their prices. The question is then how they would set the bundle price. We consider a simple structure of strategic alliance in which the participating firms independently quote for a discount for their own component in a bundle, and then, the bundle price becomes the sum of the two discounted prices. To sell a bundle, the firms need to offer a discount. Otherwise, consumers would only purchase the stand-alone products. Suppose p_b^s and p_{iA}^s, for $i = 1, 2$, are the allied firms' prices for a bundle and two stand-alone products, respectively. Let $\delta_i p_{iA}^s$ be the discounted price of firm A_i's product in a bundle, where $\delta_i \in [0, 1]$, $i = 1, 2$. Then, $p_b^s = \sum \delta_i p_{iA}$. If $\delta_i = 1$, $p_b^s = p_{1A}^s + p_{2A}^s$. In this case, there is no discount for a bundle and the price of a bundle is the sum of two stand-alone products.

Since $v_{12} \gg v_1 + v_2$, consumers always prefer buying both products to buying only one of them. Figure 1 summarizes the market demands for a bundle and stand-alone products. In Figure 1, the areas D_{bundle}^s, A_1B_2, B_1A_2, and B_1B_2 show the demands for the allied firm's bundled products, two different mix-and-match products, and the non-allied firms' products, respectively. An increase in the bundle discount (i.e., a lower δ_i) would make consumers who otherwise would have chosen to mix and match or to purchase from B_1 and B_2 switch to the allied firms' bundled products. However, we find that the optimal δ_i for the allied firms is 1, for $i =$ 1,2.

Proposition 1 *Suppose A_1 and A_2 form an alliance to offer mixed bundling. Firms do not offer a discount for a bundle, i.e. $\delta_1^* = \delta_2^* = 1$. As a result, allied firms do not gain from mixed bundling.*

Proof. *All the proofs are provided in the Appendix.* ■

Firms do not offer a bundle discount because of free-riding incentives. Suppose A_2 offers a small discount for its component in a bundle in order to increase the sales of the bundled products. Then A_1profits from the discount offered by A_2 even if it does not offer any discount for its component in a bundle. If A_2 offers no discount, A_1 does not want to offer a discount either. Thus, firms will never offer a bundle discount. As there is no discount for a bundle, the prices are the same as before bundling and the outcome under mixed bundling is equivalent to the one in the benchmark case without bundling. Thus, firms do not gain from mixed bundling.

3.1.2 Merger

Suppose A_1 and A_2 merge instead. Let $p_b^m$and p_{iA}^m for $i = 1, 2$ be the merged firm M's prices for a bundle and two stand-alone products. $\lambda_m = p_{1A}^m + p_{2A}^m - p_b^m \geq 0$ is a bundle discount offered by the merged firm. The market demands for a bundle and stand-alone products are similar to the case of strategic alliance. However, unlike the case of strategic alliance, the merged firm is now able to coordinate its stand-alone prices along with a bundle discount to optimize the use of a bundle discount. Since a small discount for a bundle λ_m increases the merged firm's market share, the merged firm is always inclined to offer a positive discount for consumers who buy a bundle. However, Proposition 1 shows that the merged firm incurs losses from the mixed bundling.

Proposition 2 *Suppose A_1 and A_2 are merged. The profits from mixed bundling are lower than the profits without bundling.*

To sell a bundle, the merged firm needs to offer a discount. Otherwise, consumers would only purchase the stand-alone products. Such a bundle discount would attract more consumers to purchase both products from the merged firm. However, since the increased market share is at a discounted price, the firm needs to make up the profits by increasing the stand-alone prices and exploiting the surplus from the consumers who highly value the mix-and-match products. In the present framework, such a strategy turns out to be unprofitable because in response to the bundle discount, rivals lower their prices. As the price competition with rivals over the markets for stand-alone products become more intense, the merged firm cannot price their stand-alone products high enough to recoup the profit. In a multi-product duopoly model with $t_1 = t_2$, Matutes and Régibeau (1992) and Armstrong (2006) show a similar result that a multi-product firm does not gain from unilateral mixed bundling. Proposition 2 states that the same result holds even when $t_2 > t_1$ and when the rivals are not merged.

In the Appendix, we show that the losses from mixed bundling are greater when $t_2 > t_1$. This is because when $t_2 > t_1$, the merged firm has to offer a higher bundle discount to make consumers switch to purchase a bundle since consumers are less inclined to shop around for the second product. A higher bundle discount will make the price competition for the stand-alone product demands even tougher, making it more difficult to profit from mixed bundling for the merged firm.

In addition, in the Appendix, from the proof of Proposition 1, we find that the free-riding incentive of allied firms in mixed bundling is independent of whether the rivals are merged or not. That is, regardless of whether A_1 and A_2 are merged, firms B_1 and B_2 will offer no discount for a bundle if they form a strategic alliance. This implies that the result in Proposition 2 holds even if B_1 and B_2 form a strategic alliance to offer mixed bundling. That is, the merged firm incurs losses from mixed bundling, regardless of whether the rivals offer mixed bundling via strategic alliance. Hence, we get the following corollary.

Corollary 1 *When a pair of firms are merged and the other pair of firms form a strategic alliance, no firms gain from mixed bundling.*

While mixed bundling is unprofitable, pure bundling can increase firms' profits. The next section shows the profitability of pure bundling under strategic alliance and merger.[6]

3.2 Strategic Alliance for Pure Bundling

Here we examine the effect of unilateral pure bundling on firms' profits, especially when firms offer pure bundling through a strategic alliance instead of a merger. Under pure bundling, the sales of individual components are not available. Firms may agree to sell their products only in bundles if it is profitable. One easily observed form of pure bundling is technological tying. Technological tying occurs when firms design their product in a way that it would function only if used with the complementary products. When a firm manufactures multiple products, it is likely that the products are technologically tied together. Naturally, it is very difficult to prohibit such a type of bundling. There has been a lenient legal standard on technological tying because any successful anticompetitive claim would require proof that the integration of the products was solely for tying itself, rather than for any technological benefits from tying.

On the other hand, tying arrangement between two complementary product producers via strategic alliance is not uncommon in practice either. For example, in 2007, upon the release of the iPhone, Apple entered an exclusive tying arrangement

[6] Most studies in the literature find that firms favor either mixed bundling over pure bundling or pure bundling over mixed bundling. By contrast, Vaubourg (2006) presents an interesting result; in equilibrium, pure bundling offered by one firm and mixed bundling offered by its rival may coexist. This explains why some firms prefer pure bundling while others prefer mixed bundling.

with AT&T, which made consumers unable to buy iPhone without subscribing to the AT&T network service. In doing so, Apple installed a technological locking device on all iPhones that blocked access to other non-AT&T networks. This tying arrangement ended in 2011 as a result of an antitrust suit. Another example of pure bundling via strategic alliance is found in the markets for 3D DVD players and 3D movies. Monsters vs Aliens, Shrek, and How To Train Your Dragon are sold exclusively only with the purchase of Samsung 3D DVD players, and Avatar, Ice Age, and Dawn of the Dinosaurs are exclusive to Panasonic players.

Suppose firms A_1 and A_2 agree to sell their products only in bundles. Let p_{AA} be the price of a bundle offered by A_1 and A_2. Since there are no separate sales of each item, by construction, $\delta_i = 1$. Then, the bundle price is determined by the sum of two stand-alone prices, i.e., $p_{AA} = p_{1A}^{sp} + p_{2A}^{sp}$, where p_{1A}^{sp} and p_{2A}^{sp} are the optimal prices under the alliance.[7] The firms pre-negotiate how to divide up the profit from pure bundling and then set their prices independently. For any pre-determined profit sharing rule (ϕ_1, ϕ_2), where $0 < \phi_i < 1$, $\sum \phi_i = 1$, the two firms abide by the agreement only if they *both* find that bundling is at least as profitable as before the alliance.

As a result of pure bundling, the two markets become interdependent. Consumers have only two choices, namely, purchasing the two products either from A_1 and A_2 or from B_1 and B_2. If A_1 and A_2 offer pure bundling, there is no difference in whether or not B_1 and B_2 counter the bundling via strategic alliance as well. If a consumer does not want to buy a bundle from A_1 and A_2, she must buy the two products from B_1 and B_2, regardless of whether their products are bundled. Moreover, even if B_1 and B_2 offer mixed bundling, as long as A_1 and A_2 offer pure bundling, then the market outcomes are the same since no consumer would be able to mix and match the products by B_1 or B_2 with the products by A_1 or A_2. Hence, the following analysis applies to all the three cases, when B_1 and B_2 offer mixed or pure bundling via strategic alliance, and when they don't.

A consumer with x_{12} buys a bundle from A_1 and A_2 if and only if

$$\begin{aligned} & p_{AA} + t_1(x_1)^2 + t_2(x_2)^2 \leq p_{1B}^{sp} + t_1(1-x_1)^2 + p_{2B}^{sp} + t_2(1-x_2)^2 \\ \Leftrightarrow \quad & x_2 \leq \alpha^s + \beta - \gamma x_1, \end{aligned} \tag{3}$$

where $\alpha^s := \frac{p_{1B}^{sp} + p_{2B}^{sp} - p_{AA}}{2t_2}$, $\beta := \frac{t_1+t_2}{2t_2}$, and $\gamma := \left(\frac{t_1}{t_2}\right) \leq 1$. We find that the equilibrium exists only if $0 < \alpha^s + \beta < 1$ and $0 < \alpha^s + \beta - \gamma < 1$.

Lemma 1 *No equilibrium exists if $\alpha^s + \beta > 1$ or $\alpha^s + \beta < \gamma$.*

Figure 2 depicts the demands for the bundle provided by A_1 and A_2 and the two stand-alone products, or another bundle, provided by B_1 and B_2, respectively, when

[7] We call this structure "independent pricing." In section 5, we briefly discuss an alternative type of strategic alliance and show that firms would prefer independent pricing structure.

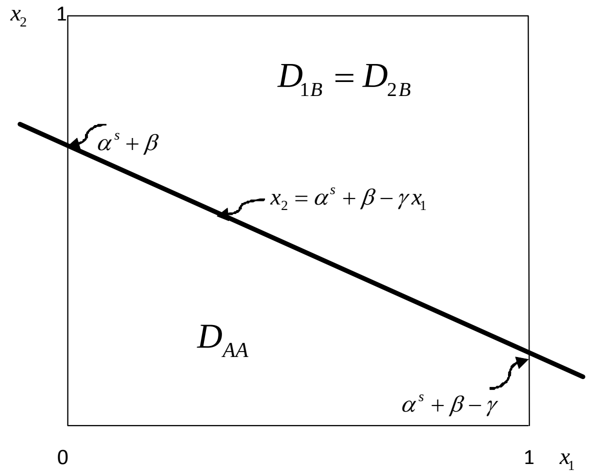

Figure 2: Demand for a Bundle: Bundling through a Strategic Alliance

$0 < \alpha^s + \beta < 1$ and $0 < \alpha^s + \beta - \gamma < 1$. The demand for a bundle is

$$D_{AA}(p_{AA}, p_{1B}^{sp}, p_{2B}^{sp}) = D_{1A}^{sp} = D_{2A}^{sp} = N\left\{\alpha^s + \beta - \frac{\gamma}{2}\right\} = N\left\{\alpha^s + \frac{1}{2}\right\}. \tag{4}$$

The demand for good i from B_i is

$$D_{iB}^{sp}(p_{AA}, p_{1B}^{sp}, p_{2B}^{sp}) = N\left\{\frac{1}{2} - \alpha^s\right\}, \text{ for } i = 1, 2.$$

The profit functions are $\pi_{iA}^{sp} = \phi_i D_{AA}(p_{AA} - c_1 - c_2) - f_i$ and $\pi_{iB}^{sp} = D_{iB}^{sp}(p_{iB}^{sp} - c_i) - f_i$ for $i = 1, 2$.[8] Solving the first-order conditions, we get $D_{AA} = D_{iB}^{sp} = \frac{N}{2}$. The optimal prices are

$$\begin{aligned} p_{AA} &= p_{1A}^{sp} + p_{2A}^{sp} = 2t_2 + c_1 + c_2, \\ p_{1B}^{sp} &= p_{1A}^{sp} = t_2 + c_1, \text{ and} \\ p_{2B}^{sp} &= p_{2A}^{sp} = t_2 + c_2. \end{aligned}$$

The profits are

$$\pi_{iA}^{sp} = \phi_i N t_2 - f_i. \tag{5}$$

$$\pi_{iB}^{sp} = \frac{N}{2} t_2 - f_i, \ i = 1, 2. \tag{6}$$

[8]If allied firms sell their products independently as in the example of video games and game consoles, the profit functions for the allied firms are $\pi_{iA}^{sp} = D_{AA}(p_{iA}^{sp} - c_i) - f_i$. This is a special case of the general framework for pure bundling described in this section.

From (2) and (5), it is straightforward to see that $\sum_i \pi_{ij}^{sp} > \sum_i \pi_{ij}^{*}$, for all $j = A, B$.

Also, $\pi_{iA}^{sp} > \pi_{iA}^{*}$ for all $i = 1, 2$ if $\phi_2 > \frac{1}{2}$ and $\phi_1 = 1 - \phi_2 > \frac{t_1}{2t_2}$. That is, if $\frac{1}{2} < \phi_2 < 1 - \frac{t_1}{2t_2}$, both firms gain from the strategic alliance. Such (ϕ_1, ϕ_2) always exists if $t_2 > t_1$.

Proposition 3 *Suppose A_1 and A_2 form an alliance to bundle their products. Pure bundling is always profitable if $t_2 > t_1$.*

As a result of pure bundling, all firms, including the firms that do not offer bundling, are at least weakly better off. This is because pure bundling enhances the market power of firms by removing consumers' ability to mix and match the two complementary products. As the two products are inseparable, consumers face fewer choices than before, and thus, they have limited ability to shop around among firms. While the transportation cost for good 1 (t_1) is lower than the one for good 2 (t_2), A_1 can now charge $t_2 = \max\{t_1, t_2\}$ for good 1 as a component of a bundle, because consumers can no longer purchase one product without the other. This way, pure bundling expands the market power of the firms.

Consider the example of tying between 3D DVD movies and the player. Suppose consumers have strong preferences for particular movies while they might not be as picky about the brand of the players. If players are not bundled with the movies, the prices for the players would have been much more competitive since consumer preferences for the brand of player are not as strong as for movies. However, as the players are now sold only in a bundle with the movies, firms know that consumers will choose one bundle over the other most likely based on the movies. That is, if a consumer buys a bundle with a Samsung player instead of a bundle with a Panasonic, it is most likely because she likes Shrek more than Avatar. Then, firms know that even if they charge a little more for their player, they would not lose a lot of demand because consumers with a strong preference over the movie with which the player is bundled would still buy that bundle instead of the competitor's bundle. As a result, the price of a bundle is higher than what the firms would have charged for the two when they were sold separately. Hence, bundling the players with the movies that consumers are picky about, firms gain more market power over the sales of the players.

3.3 Merger for Pure Bundling

Suppose now A_1 and A_2 merge in order to bundle. The merged firm M offers pure bundling for the two products at the price of p_M. Let p_{1B}^p and p_{2B}^p be the prices of rivals, B_1 and B_2, respectively. Consumers can buy both goods from M, or they can buy good 1 and good 2 from B_1 and B_2 separately.[9] A consumer with x_{12} buys both

[9]The fact that consumers cannot mix and match one of the unmerged firms' stand-alone products with one of the merged firm's bundled products implies that under pure bundling, the merged firm's

goods from M if and only if

$$\begin{aligned} & p_M + t_1(x_1)^2 + t_2(x_2)^2 \leq p_{1B}^p + t_1(1-x_1)^2 + p_{2B}^p + t_2(1-x_2)^2 \\ \Leftrightarrow \quad & x_2 \leq \alpha^M + \beta - \gamma x_1, \end{aligned} \tag{7}$$

where $\alpha^M := \frac{p_{1B}^p + p_{2B}^p - p_M}{2t_2}$.

In the Appendix, we show that the merge-and-bundle strategy can be profitable only in the range where $\alpha^M + \beta < 1$, i.e., $p_{1B}^p + p_{2B}^p - p_M < t_2 - t_1$. In this case, the demand for a bundle is $D_M^p(p_M, p_{1B}^p, p_{2B}^p) = N\left\{\alpha^M + \frac{1}{2}\right\}$ and the demand for an individual product i is $D_{iB}^p(p_M, p_{1B}^p, p_{2B}^p) = N\left\{\frac{1}{2} - \alpha^M\right\}$, for $i = 1, 2$. The profit functions are $\pi_M^p = N\left\{\alpha^M + \frac{1}{2}\right\}(p_M - (c_1 + c_2)) - \sum f_i$ and $\widetilde{\pi}_{iB}^p = N\left\{\frac{1}{2} - \alpha^M\right\}(p_{iB}^p - c_i) - f_i$ for $i = 1, 2$. Solving the first-order conditions, we obtain $\alpha^M + \frac{1}{2} = \frac{5}{8}$,and thus, $D_M^p = \frac{5}{8}N$ and $D_{iB}^p = \frac{3}{8}N$. The optimal prices are

$$\begin{aligned} p_M &= \frac{5}{4}t_2 + c_1 + c_2 < p_{1B}^p + p_{2B}^p, \\ p_{iB}^p &= \frac{3}{4}t_2 + c_i, \text{ for } i = 1, 2. \end{aligned}$$

These prices satisfy the condition $\alpha^M + \beta < 1$ only if $t_1 < \frac{3}{4}t_2$. The profits are

$$\begin{aligned} \pi_M^p &= \frac{25N}{32}t_2 - \sum f_i, \\ \widetilde{\pi}_{iB}^p &= \frac{9N}{32}t_2 - f_i, \text{ for } i = 1, 2. \end{aligned} \tag{8}$$

Proposition 4 *Suppose A_1 and A_2 are merged. If $t_1 < \frac{9}{16}t_2$, the merged firm's profit increases from pure bundling.*

While pure bundling softens competition, the merged firm realizes that internalizing the complementarity between the two products (the Cournot effect), they can price more aggressively to increase their market share. The merged firm offers a discount for a bundle, i.e., $p_M < p_{1B}^p + p_{2B}^p$, which increases the merged firm's market share, i.e., $D_M^p = \frac{5}{8}N > \frac{1}{2}N$. As a result, merger entails more intense price competition with rivals. The merger may not even be profitable in some parameter ranges. Because the merged firm gains $\frac{1}{8}N$ of extra market share by charging the two

products become incompatible with rivals' unbundled products. In many cases, pure bundling imposes such incompatibility. But in some cases, unmerged rivals may have choices over whether to make their products incompatible with the merged firm's bundled products. For example, in the case of tying between game consoles and disc formats, when Sony announced to use Blu-ray disc for Playstation 3, Microsoft could have chosen Blu-ray as well for Xbox 360 consoles.

Denicolo (2000) models whether the rivals (specialists) have incentives to make their products incompatible in this context. He shows that if the degree of product differentiation is high, as in the current framework, incompatibility arises in equilibrium.

products at $\frac{5}{4}t_2$ whereas the pre-bundling prices are t_1+t_2, whether or not the merge-and-bundle strategy improves profits depends on the size of the taste parameters t_2 and t_1. If t_1 is much smaller than t_2, i.e., $t_1 < \frac{9}{16}t_2$, then the losses from a discount are small compared to the gain from a higher market share; thus, bundling through a merger is profitable.

However, since the profitability arises from an aggressive price cut, bundling through a merger is not as profitable as bundling through a strategic alliance. Comparing the profits from (5) and (8), we obtain

$$\pi_M^p < \pi_{1A}^{sp} + \pi_{2A}^{sp}.$$

Corollary 2 *Bundling through a strategic alliance is more profitable than bundling through a merger.*

When A_1 and A_2 merge to bundle, as long as the merger is profitable, i.e., $t_1 < \frac{9}{16}t_2$, B_1benefits from the merger in the short run[10], since p_{1B}^p increases if $t_1 < \frac{3}{4}t_2$. By contrast, B_2 always incurs losses as a result of the merger, i.e., $\pi_{2B}^p = \frac{9}{32}t_2N - f_2 < \pi_{2B}^* = \frac{1}{2}t_2N - f_2$. While a merger is immediately not as profitable as a strategic alliance for A_1 and A_2, the fact that one of the rival firms incurs losses due to the merger becomes an important motivation for merger, since the merger will be more profitable in the long run if it induces foreclosure. But what is more interesting is that this possibility of foreclosure also motivates a strategic alliance between B_1 and B_2. Next, we discuss the incentives for a counter-bundling in this context.

3.4 Incentives for Strategic Alliance to Counter Bundling

Consider the unmerged rival firms' incentive to form an alliance to offer pure bundling in order to compete against the merged firm M. We have two possible cases: (i) when both M and allied rivals offer pure bundling and (ii) when the allied rivals alone offer pure bundling.

First, consider the case when both M and the allied rivals offer pure bundling. Let $p_{BB} = p_{1B}^p + p_{2B}^p$ be the price of a bundle offered by B_1 and B_2. A consumer with x_{12} buys both goods from M if and only if (7) is satisfied. The demand for the merged firm's bundle is $D_M^p(p_M, p_{BB}) = N\left\{\widetilde{\alpha}^M + \frac{1}{2}\right\}$ and the demand for the allied firms' bundle is $D_{BB}^p(p_M, p_{BB}) = N\left\{\frac{1}{2} - \widetilde{\alpha}^M\right\}$, where $\widetilde{\alpha}^M := \frac{p_{BB}-p_M}{2t_2}$. The profit functions are $\pi_M^p = N\left\{\widetilde{\alpha}^M + \frac{1}{2}\right\}(p_M - (c_1+c_2)) - \sum f_i$ and $\pi_{iB}^p = N\left\{\frac{1}{2} - \widetilde{\alpha}^M\right\}(p_{iB}^p - c_i) - f_i$ for $i = 1, 2$. The optimal prices are $p_M = \frac{5}{4}t_2 + \sum c_i$ and $p_{BB} = \frac{6}{4}t_2 + \sum c_i$. The profits are

$$\begin{aligned} \pi_M^p &= \frac{25N}{32}t_2 - \sum f_i, \text{ and} \\ \pi_{iB}^p &= \phi_{iB}\frac{9N}{16}t_2 - f_i, \end{aligned} \tag{9}$$

[10] If merger leads to foreclosure of B_2, this benefit will not last in the long run. We discuss this case in the next subsection.

where $0 < \phi_{iB} < 1$ is the pre-negotiated profit share of B_i, and $\sum \phi_{iB} = 1$, for $i = 1, 2$. Thus, M profits from pure bundling if $t_1 < \frac{9}{16}t_2$.

Now consider the case when the allied rivals alone offer pure bundling. Let $\widehat{p_{1A}}$, $\widehat{p_{2A}}$, and $\widehat{p_{BB}}$ be the prices of M's individual products in market 1 and market 2 and the allied rivals' bundle price, respectively. By construction, $\widehat{p_{BB}} = \widehat{p_{1B}} + \widehat{p_{2B}}$. A consumer with x_{12} buys the two products separately from M if and only if $x_2 \leq \widehat{\alpha} + \beta - \gamma x_1$, where $\widehat{\alpha} := \frac{\widehat{p_{BB}} - (\widehat{p_{1A}} + \widehat{p_{2A}})}{2t_2}$. The demand for stand-alone products produced by M is $\widehat{D_M}(\widehat{p_{1A}}, \widehat{p_{2A}}, \widehat{p_{BB}}) = N\left\{\widehat{\alpha} + \frac{1}{2}\right\}$, and the demand for a bundle is $\widehat{D_{BB}}(\widehat{p_{1A}}, \widehat{p_{2A}}, \widehat{p_{BB}}) = N\left\{\frac{1}{2} - \widehat{\alpha}\right\}$. We obtain $\widehat{p_{1A}} + \widehat{p_{2A}} = p_M = \frac{5}{4}t_2 + c_1 + c_2 < \widehat{p_{BB}} = \sum p^p_{iB} = \frac{6}{4}t_2 + c_1 + c_2$. The profits are

$$\begin{aligned}
\widehat{\pi_M} &= \sum \widehat{\pi_{iA}} = \frac{25}{32}t_2 N - (f_1 + f_2) = \pi^p_M, \text{ and} \\
\widehat{\pi_{iB}} &= \phi_{iB}\frac{9N}{16}t_2 - f_i = \pi^p_{iB}, \text{ for } i = 1, 2.
\end{aligned}$$

Thus, the merged firm's profit improves from the allied firms' pure bundling. Proposition 5 summarizes the results.

Proposition 5 *Suppose A_1 and A_2 are merged.*

1. *Independently of whether B_1 and B_2 form an alliance to counter bundling, if $t_1 < \frac{9}{16}t_2$, the merged firm profits from pure bundling. Firm B_i earns $\pi^p_{iB} = \phi_{iB}\frac{9N}{16}t_2 - f_i$ from a strategic alliance with B_{-i}, and $\widetilde{\pi}^p_{iB} = \frac{9N}{32}t_2 - f_i$ without alliance, for $i = 1, 2$.*

2. *If $\frac{16}{9}t_1 < t_2 < \frac{32}{9N}f_2$, as long as $\phi_{1B} \geq \frac{16}{9t_2 N}f_1$ and $\phi_{2B} = 1 - \phi_{1B} \geq \frac{16}{9N}t_2 f_2$, B_1 and B_2 can evade foreclosure via a strategic alliance, where ϕ_{iB} is the profit share of B_i under the alliance.*

3. *If $\frac{t_1 t_2 N}{(f_1+f_2)} < \frac{16}{9}t_1 < t_2 < \frac{32}{9N}f_2$, the strategic alliance between B_1 and B_2 fails, and thus, the merger leads to the foreclosure of B_1 and B_2.*

If A_1 and A_2 are merged, B_1 and B_2 cannot improve their profits by counter-bundling with alliance. Similar to the case in section 3.2, once the merged firm sells the two products only in a bundle, regardless of whether B_1 and B_2 bundle their products as well or not, consumers face the same two choices: purchasing the two products either from A_1 and A_2 or from B_1 and B_2. What is different now is that one pair of firms are merged and the others are not. The merged firm can coordinate the two complementary product prices and thus aggressively price its products while allied firms cannot. Since counter-bundling does not enhance the allied firms' market power or make them able to internalize the complementarity of the two products, there would be no impact on the allied firms' profits. This structural difference between strategic alliance and merger becomes more evident when the allied firms alone offer pure bundling while the merged firm does not offer bundling at all. The first result in Proposition 5 states that B_1 and B_2 do not gain from strategic alliance even in this case. That is, even when it is not bundling, the merged firm will profit from rivals' pure bundling if the rivals are not merged. The allied firms do not gain from bundling since only the merged firm gets the advantage of internalizing the complementarity of the two products.

Nevertheless, B_1 and B_2 may still want to form an alliance when A_1 and A_2 are merged because they can avoid foreclosure with profit sharing under the alliance. In the range where $\frac{16}{9}t_1 < t_2 < \frac{32}{9N}f_2$, merger is profitable and, without alliance, B_2 incurs losses, i.e., $\widetilde{\pi}_{2B}^p = \frac{9N}{32}t_2 - f_2 < 0$.[11] As B_2 becomes non-viable, without a strategic alliance, foreclosure is expected. If B_2 exits, then B_1 will be also forced to withdraw from the market since there will be no demand for B_1's product separately. Knowing this, B_1 may be willing to share its profits with B_2 in order to make B_2 viable so that they can avoid foreclosure. To be viable, B_2 must earn $\phi_{2B}\frac{9N}{16}t_2 \geq f_2$. Thus, it must be that $\phi_{2B} = 1 - \phi_{1B} \geq \frac{16}{9t_2N}f_2$. B_1 will be still making profits if $\phi_{1B} \geq \frac{16}{9t_2N}f_1$. If $t_2 > \frac{16}{9N}(f_1 + f_2)$, such a profit sharing rule (ϕ_{1B}, ϕ_{2B}) exists, and B_1 and B_2 can evade foreclosure with a strategic alliance.

Naturally, the alliance between B_1 and B_2 breaks down if B_1 incurs losses for any (ϕ_{1B}, ϕ_{2B}). The lowest profit share that makes B_2 viable is $\underline{\phi_{2B}} = 1 - \overline{\phi_{1B}} = \frac{16}{9t_2N}f_2$. At such a $\overline{\phi_{1B}}$, B_1 incurs losses if $\overline{\phi_{1B}} = 1 - \frac{16}{9t_2N}f_2 < \frac{16}{9t_2N}f_1$, i.e., $t_2 < \frac{16}{9N}(f_1 + f_2)$. Intuitively, this is the case when the B_1's profit is not big enough to cover B_2's losses, i.e., $t_2 - \frac{32}{9N}f_1 < \frac{32}{9N}f_2 - t_2$. In this case, alliance does not arise, and thus, merger induces foreclosure. Therefore, if $\frac{16}{9}t_1 < t_2 < \frac{32}{9N}f_2$ and $t_2 < \frac{16}{9N}(f_1 + f_2)$, merger leads to successful foreclosure. Combining the two conditions and rearranging terms, we get $\frac{t_1 t_2 N}{(f_1+f_2)} < \frac{16}{9}t_1 < t_2 < \frac{32}{9N}f_2$ for this case.

The results in Proposition 5 imply that merger is less likely to facilitate exclusionary bundling if unmerged rival firms can counter bundling through a strategic alliance. As foreclosure is less likely to occur, A_1 and A_2 would be less interested in merger, given that without foreclosure, merger is less profitable than strategic alliance (Corollary 1). Hence, the possibility of strategic alliance to counter bundling reduces antitrust concerns for the type of merger that facilitates exclusionary bundling.

[11] We do not consider the possibility of predatory bundling in this paper. That is, if merger is unprofitable prior to foreclosure, firms will not have an incentive to merge and bundle just to foreclose competition. Nalebuff (2005) distinguishes exclusionary bundling from predatory bundling in the following sense: "Exclusionary bundling has a foreclosure effect similar to that of predatory pricing, but the two practices have important differences. Unlike traditional predatory pricing, the exclusionary behavior need not be costly to the firm. (p.321)"

If A_1 and A_2 merge		B_1 and B_2		
		Mixed Bundling (SA)	Pure bundling (SA)	Do not bundle
Merged Firm	Mixed bundling	π_M^{m*} π_{iB}^{m}	π_M^{P} π_{iB}^{P}	π_M^{m*} π_{iB}^{m}
	Pure bundling	π_M^{P} π_{iB}^{P}	π_M^{P} π_{iB}^{P}	π_M^{P} $\tilde{\pi}_{iB}^{P}$
	Do not bundle	π_M π_{1B}^{*}	π_M^{P} π_{iB}^{P}	π_M π_{iB}^{*}

where (1) $\pi_M^P = \frac{25N}{32}t_2 - \sum f_i > \pi_M = \sum \pi_{iA}^* = \frac{N}{2}\sum t_i - \sum f_i$ if $\frac{16}{9}t_1 < t_2$, (2) $\sum \pi_{iA}^* > \pi_M^{m*}$

(3) $\pi_{iB}^P = \phi_{iB}\frac{9N}{16}t_2 - f_i$, $\tilde{\pi}_{iB}^P = \frac{9N}{32}t_2 - f_i$, (4) $\pi_{iB}^* = \pi_{iA}^* = \frac{N}{2}t_i - f_i$, i=1,2.

If A_1 and A_2 do not merge		B_1 and B_2		
		Mixed Bundling (SA)	Pure bundling (SA)	Do not bundle
A_1 and A_2	Mixed bundling (SA)	π_{iA}^{*} π_{iB}^{*}	π_{iA}^{sp*} π_{iB}^{sp*}	π_{iA}^{*} π_{iB}^{*}
	Pure bundling (SA)	π_{iA}^{sp*} π_{iB}^{sp*}	π_{iA}^{sp*} π_{iB}^{sp*}	π_{iA}^{sp*} $\tilde{\pi}_{iB}^{sp*}$
	Do not bundle	π_{iA}^{*} π_{iB}^{*}	π_{iA}^{s*} π_{iB}^{sp*}	π_{iA}^{*} π_{iB}^{*}

where (1) $\pi_{iA}^{sp*} = \phi_{iA}Nt_2 - f_i > \pi_{iA}^*$, $\pi_{iB}^{sp*} = \phi_{iB}Nt_2 - f_i > \pi_{iB}^*$,

(2) $\pi_{iA}^{s*} = \frac{N}{2}t_2 - f_i \geq \pi_{iA}^*$, (3) $\tilde{\pi}_{iB}^{sp*} = \frac{N}{2}t_2 - f_i \geq \pi_{iB}^*$

Figure 3: Payoffs in the subgames for a unilateral merger between A_1 and A_2

3.5 Incentives for Merger

Now we analyze the merger incentives for A_1 and A_2 at Stage 0. The possibility of counter-merger by B_1 and B_2 will be incorporated in the next section. Combining the results from sections 3.1 through 3.4, we obtain the following predictions for a game where a unilateral merger decision between A_1 and A_2 is considered.

Proposition 6 *Consider a game where A_1 and A_2 are deciding to merge.*

1. *If $\frac{t_1t_2N}{(f_1+f_2)} < \frac{16}{9}t_1 < t_2 < \frac{32}{9N}f_2$, A_1 and A_2 merge and offer pure bundling to foreclose competition.*

2. *Otherwise, A_1 and A_2 do not merge, and firms offer pure bundling through strategic alliances.*

Figure 3 summarizes the payoffs in each subgame. If A_1 and A_2 merge, the merged firm offers pure bundling, since pure bundling is a dominant strategy for the merged firm irrespective of rivals' strategy choices, and thus, the outcomes in this subgame always involve pure bundling by the merged firm.[12] In the range where $\frac{t_1t_2N}{(f_1+f_2)} < \frac{16}{9}t_1 < t_2 < \frac{32}{9N}f_2$, rivals cannot avert foreclosure with strategic alliance.

[12] Multiple outcomes are possible. However, the payoffs in all of the cases are identical for the merged firm. The payoffs for rival firms depend on whether they form an alliance. If they do, the final profit will be dependent on the bargaining solution for profit-sharing. See Figure 3 for details.

Thus, the merged firm earns π_M^p and enjoys the monopoly profit after foreclosure. [13] While π_M^p is lower than the profit under strategic alliance, $\sum \pi_{iA}^{sp*}$, combined with foreclosure profits, merger becomes more profitable. Thus, at stage 0, A_1 and A_2 merge and bundle. In other ranges of parameters, however, either merger is not profitable, or merger is profitable but foreclosure is not possible. Thus, merger does not occur and A_1 and A_2 choose strategic alliance. Firms profit from pure bundling via alliance since the price of good 1 increases from $t_1 + c_1$ to $t_2 + c_1$. Hence, the profits are at the expense of consumer surplus. In addition, consumers incur extra welfare cost since some of them have to incur a lot of transportation costs as they are not allowed to mix and match.

In the present framework, mergers and strategic alliances result in different pricing strategies for firms because merging partners coordinate their prices to bundle as they become completely integrated after merger, whereas under a strategic alliance, allied firms independently set their own prices. However, as it is a matter of choice for a merged firm to decide how to reorganize its production structure after merger, one might wonder whether a merged firm would have an incentive to fully integrate the merging partners in the current framework. A few studies have shown that firms may have strategic incentives to keep competition within the operating units. For example, Baye, Crocker, and Ju (1996) model firms' incentives to divide production among autonomous competing units through divisionalization, franchising, or divestiture. Mialon (2008) shows that merged firms may prefer retaining competition between the merging partners because such a structure promotes efficiency-improving capital reallocation between the merging partners. In the present model, if a merged firm chooses to keep competition within the merging partners, the resulting structure resembles that of a strategic alliance. Given that merger is only motivated by foreclosure possibility, when merger occurs, (i.e., when $\frac{t_1 t_2 N}{(f_1+f_2)} < \frac{16}{9} t_1 < t_2 < \frac{32}{9N} f_2$), it would not be optimal for the merged firm to operate on autonomous divisions because without full integration, the merged firm cannot induce foreclosure. In other ranges of parameters, there is no incentive for merger. Thus, when merger occurs, firms always choose to fully integrate the merging partners.

[13] While the decision for A_1 and A_2 to merge at stage 0 is dependent on the long-term profitability of foreclosure, Figure 3 does not include the payoff of the merged firm after foreclosure. This is because for a full description of profits for the merged firm after foreclosure, we would need to introduce a new game between the merged firm and *new rivals* (potential entrants) in a later period; this involves completely different competition and market parameters such as entry costs E_i of a potential entrant b_i in market $i, i = 1, 2$. However, to justify the current reduced form, we only need to show that the merged firm's profit after foreclosure is higher than the profits from bundling through a strategic alliance.

While this is somewhat straightforward, in the Supplementary Appendix, we provide a full description of the payoffs for this extended game after foreclosure, and show that foreclosure after merger is more profitable than bundling via strategic alliance.

4 Equilibrium Merger and Strategic Alliance

In this section, we derive the equilibrium of the game where firms simultaneously decide whether to merge and bundle. Without loss of generality, we assume that the possibility of merger is discussed between B_1 and B_2 as well as between A_1 and A_2. That is, at Stage 0, two pairs of firms, (A_1,A_2) and (B_1,B_2), simultaneously decide whether to merge and at Stage 1, each pair of firms decides whether to bundle and how to bundle.

Lemma 2 *If both pairs of firms merge, neither mixed bundling nor pure bundling is profitable for firms.*

If both pairs of firms merge, it is never profitable to offer mixed bundling. See Armstrong (2006) for the explanation. We find that pure bundling is also unprofitable if it is offered by both merged firms. Suppose $A_1$and A_2 merge to offer pure bundling. If B_1 and B_2 also merge, from the Appendix, the merged entity of B_1 and B_2 earn $\pi^p_{MB} = \frac{1}{2}t_2N - (f_1 + f_2)$, which is much lower than the pre-merger profits $\sum \pi^*_{iB} = \frac{1}{2}(t_1 + t_2)N - (f_1 + f_2)$. The losses for B_1 and B_2 are much larger than what they would have had if they had not merged, i.e., $\pi^p_{MB} < \sum \pi^p_{iB} = \frac{9N}{16}t_2 - (f_1 + f_2)$. Moreover, the merger between $A_1$and A_2 also incurs the same losses, i.e., $\pi^p_{MA} = \pi^p_{MB}$. This is because as a result of the counter-merger, price competition between the two merged firms becomes more intense. As both merged firms are now able to internalize the complementarity between the two products, the two merged firms aggressively cut their prices to win the market but end up incurring severe losses with no gain in market share. The same result happens even if B_1 and B_2 do not bundle, because what triggers the attritious price war is the counter-merger, not the counter-bundling. Thus, if both pairs of firms merge, neither of the merged firms will survive in equilibrium when either of them offers bundling. If neither offers bundling, bilateral mergers result in the same outcome as prior to bundling.

When both pairs of firms merge, there are two Nash equilibria in the subgame: (pure bundling, pure bundling) and (not bundle, not bundle). Eliminating weakly dominated strategies, we obtain (not bundle, not bundle) as the unique prediction in the subgame. The equilibrium of the game is summarized in the following Proposition.

Proposition 7

1. *If $\frac{t_1t_2N}{(f_1+f_2)} < \frac{16}{9}t_1 < t_2 < \frac{32}{9N}f_2$, the unique subgame perfect Nash equilibrium of the game is that both pairs of firms merge and never bundle.*
2. *Otherwise, the unique extensive-form trembling hand perfect Nash equilibrium (THPNE) of the game is that both pairs of firms only bundle through strategic alliances.*

When $\frac{t_1t_2N}{(f_1+f_2)} < \frac{16}{9}t_1 < t_2 < \frac{32}{9N}f_2$, the unique equilibrium shows the outcome of the Prisoner's Dilemma. The dominant strategy for firms is to merge. If only one pair of firms merge and bundle, the other unmerged firms would be in danger of being forced to leave the market. Thus, both pairs of firms choose to merge. But if both pairs merge, bundling would only result in price war between the two merged firms. Hence, in equilibrium, none of them decides to bundle after observing that both mergers have occurred. However, in all other ranges of parameters, a merger is never a dominant strategy, since it is either unprofitable or sub-optimal. In equilibrium, no merger occurs, but firms offer bundling through strategic alliances.

Several studies report the outcome of the Prisoner's Dilemma in the framework of oligopoly bundling, implying that only unilateral bundling generates profits and equilibrium bundling is unprofitable in oligopoly. See, for example, Seidmann (1991), Economides (1993), and Armstrong (2006). In contrast, in this paper, the Prisoner's Dilemma arises only when foreclosure is possible. In a wide range of parameters, the equilibrium exhibits bundling via strategic alliances. Moreover, as shown in section 3.4, counter-bundling via strategic alliance reduces the range of successful foreclosure, limiting the cases where the Prisoner's Dilemma arises.

In equilibrium where bundling is offered, while firms are better off, consumers are worse off since the profit increases are due to reduced competition and price increase.

5 Discussion

In this section, we briefly discuss an alternative form of strategic alliance and firms' incentive to deviate from a strategic alliance.

5.1 Alternative Form of Strategic Alliance

In this paper, we impose an assumption that under strategic alliance allied firms' pricing decisions remain completely independent (hereafter, independent pricing) even after they agree to sell their products only in a bundle. On the other hand, if firms are able to agree on selling the products together, firms may be able to negotiate on how to set the price of a bundle along with how they divide up the profits. In this section, we discuss this alternative form of alliance and show that firms are better off with independent pricing.

Consider an alternative form of strategic alliance under which two firms, A_1 and A_2, pre-negotiate how to divide up the profit from pure bundling and then set the bundle price p_{AA} to maximize the possible profit under pure bundling. In this case, for any pre-determined profit-sharing rule (ϕ_1^c, ϕ_2^c), where $0 < \phi_i^c < 1$, $\sum \phi_i^c = 1$, the allied firms behave like a merged firm in Section 3.3 and each firm earns $\pi_{iA}^c = \phi_i^c \frac{25N}{32} t_2 - f_i$. However, unlike a merger, for allied firms the distribution of total profit matters. (ϕ_1^c, ϕ_2^c) depends on each firm's bargaining power. Suppose there is an optimal bargaining solution (ϕ_1^c, ϕ_2^c) at which each firm earns a higher profit than the profit without bundling. On the other hand, let $\widetilde{\phi_1^I}$ be the profit share of firm A_1 under independent pricing at which A_1 is indifferent between the two types of strategic alliances. Then, by construction, A_1 must earn the same profits either under independent pricing with $\widetilde{\phi_1^I}$ or under the alternative type of alliance with ϕ_1^c. That is, $\pi_{1A}^I(\widetilde{\phi_1^I}) = \widetilde{\phi_1^I} N t_2 - f_1 = \phi_1^c \frac{25N}{32} t_2 - f_1 = \pi_{A1}^c(\phi_1^c)$, implying that $\widetilde{\phi_1^I} = \phi_1^c \frac{25}{32}$. Such a $\widetilde{\phi_1^I}$ exists. Now consider $\phi_1^I = \widetilde{\phi_1^I} + \varepsilon$, $\varepsilon > 0$. We can show that for a small enough $\varepsilon > 0$, $\pi_{iA}^I(\phi_1^I) > \pi_{iA}^c(\phi_1^c)$, for all $i = 1, 2$. It is obvious that $\pi_{1A}^I(\phi_1^I) > \pi_{1A}^I(\widetilde{\phi_1^I}) = \pi_{1A}^c(\phi_1^c)$. For A_2, $\pi_{2A}^I(\widetilde{\phi_1^I} + \varepsilon) = (1 - \phi_1^c \frac{25}{32} - \varepsilon) N t_2 - f_2 > \pi_{2A}^c(\phi_1^c) = (1 - \phi_1^c) \frac{25N}{32} t_2 - f_2$ if and only if $\frac{7}{32} > \varepsilon > 0$. Thus, all firms are better off under independent pricing. Similarly, if we define $\widetilde{\phi_2^I}$ as the profit share of firm A_2 under independent pricing at which A_2 is indifferent between the two types of strategic alliances, we can show that there exists $(1 - \phi_2^I, \phi_2^I)$ at which both firms are better off under independent pricing than under the alternative type of strategic alliance, where $\phi_2^I = \widetilde{\phi_2^I} + \varepsilon$, $\frac{7}{32} > \varepsilon > 0$. Therefore, if firms can choose between the two types of strategic alliance, firms will choose independent pricing.

5.2 Stability of Strategic Alliance

Strategic alliance breaks down if one of the allied firms finds an incentive to deviate from the alliance. Since the provision of pure bundling requires that both firms agree not to sell their products individually, one may suspect stability of such an agreement between firms. We can easily find that in equilibrium, a unilateral deviation from the agreement is not profitable for firms. In order for such a deviation to be profitable, firms must be able to sell their products individually after deviation. However, in equilibrium, if no merger occurs, both pairs of firms offer pure bundling via strategic alliances. Thus, for any firm, secretly cutting its own price would not create an extra demand for its product since consumers cannot find a compatible, second product in the market, as no products are sold individually in equilibrium. Therefore, in equilibrium, strategic alliance is stable.

6 Conclusion

This paper investigates firms' incentives for merger and strategic alliance in order to bundle their product with another complementary product. To motivate antitrust concerns for a merger that induces anticompetitive bundling, the model is situated in the case where merger does not create synergy, and combined with bundling, a unilateral merger leads to foreclosure of competition. In this case, there is a range of parameters where firms choose a merger over a strategic alliance with an intention to use bundling to foreclose competition. However, the range reduces as unmerged rivals can use counter-bundling via strategic alliance to evade foreclosure. Even if an alliance to counter bundling cannot successfully avert foreclosure, in equilibrium, mergers do not lead to foreclosure since a counter-merger makes bundling unprofitable for the merged firms and thus bundling is never offered. Thus, a merger facilitating anticompetitive bundling is unlikely to arise in equilibrium in the current framework. We find that in equilibrium, firms offer pure bundling only through strategic alliances and the alliances are stable. The equilibrium prices of the two products are higher after pure bundling, and thus, consumers are worse off.

References

[1] Adams, W. J. and J. L. Yellen, 1976, "Commodity Bundling and the Burden of Monopoly" *Quarterly Journal of Economics*, 90, August, 475-498.

[2] Armstrong, M., 1999. "Price Discrimination by a Many-Product Firm," *Review of Economic Studies*, 66, 151-68.

[3] Armstrong, M., 2006, "Recent Development in the Economics of Price Discrimination," *Advances in Economics and Econometrics: Theory and Applications.*

[4] Armstrong, M. and J. Vickers, 2010, "Competitive Non-linear Pricing and Bundling," *Review of Economic Studies.*

[5] Baye, M., K. Crocker, and J. Ju, 1996. "Divisionalization, franchising, and divestiture incentives in oligopoly," *American Economic Review*, 86, 223-36.

[6] Carlton, D. & M. Waldman, 2002. "The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries," *Rand Journal of Economics*, 33(2), 194-220.

[7] Carbajo J., D. De Meza and D. Seidmann, 1990, "A Strategic Motivation for Commodity Bundling," *Journal of Industrial Economics,* 38, 283-298.

[8] Chen, Y., 1997. "Equilibrium Product Bundling," *Journal of Business*, 70: 85-103.

[9] Choi, J. P., 2008, "Mergers with Bundling in Complementary Markets," *Journal of Industrial Economics*, 56 (3), 553-577.

[10] Choi, J. P. and C. Stefanadis, 2001, "Tying, Investment, and the Dynamic Leverage Theory," *RAND Journal of Economics*, 32 (1), 52–71.

[11] DeGraba, P., 1994, "No Lease is Short Enough to Solve the Time Inconsistency Problem," *Journal of Industrial Economics*, 42, 361-374.

[12] Department of Justice, 2001, "Range Effects: The United States Perspective," Antitrust Division Submission for OECD Roundtable on Portfolio Effects in Conglomerate Mergers

[13] Denicolo, V., 2000, "Compatibility and Bundling with Generalist and Specialist Firms," *Journal of Industrial Economics*, 48(2), 177-188.

[14] Economides, N., 1993, "Mixed Bundling in Duopoly," Stern School of Business Discussion Paper EC-93-29, New York University, New York, N.Y, U.S,A.

[15] Gans, J. and S. King, 2006, "Paying for Loyalty: Product Bundling in Oligopoly," *Journal of Industrial Economics*, 54(1), 43-62.

[16] Hewitt, Gary, 2002, "Portfolio Effects in Conglomerate Mergers," OECD., Best Practice Roundtables in Competition Policy No. 37. Available at SSRN: http://ssrn.com/abstract=318777.

[17] Matutes, C. and P. Régibeau, 1992, "Compatibility and Bundling of Complementary Goods in a Duopoly," *Journal of Industrial Economics*, 40 (1), 37-54.

[18] McAfee, P. R., J. McMillan and M. D., Whinston, 1989, "Multiproduct Monopoly, Commodity Bundling, and Correlation of Values," *Quarterly Journal of Economics*, 104, May, 371-384.

[19] McCain, R. A., 1987, "Scalping: Optimal Contingent Pricing of Performances in the Arts and Sports," *Journal of Cultural Economics*, 11(1), June, 1-21.

[20] Mialon, S. H., 2008, "Efficient horizontal mergers: The Effects of Internal Capital Reallocation and Organizational Form," *International Journal of Industrial Organization*, 26, 861–877.

[21] Nalebuff, B., 2004, "Bundling as an Entry Barrier," *Quarterly Journal of Economics*, February, 119(1), 159-187.

[22] Nalebuff, B., 2005, "Exclusionary Bundling," *The Antitrust Bulletin*, 50 (3), 321-370.

[23] Peitz, M., 2008, "Bundling may blockade entry," *International Journal of Industrial Organization*, 26(1), 41-58.

[24] Seidmann, D. J., 1991, "Bundling as a Facilitating Device: A Reinterpretation of Leverage Theory," *Economica*, 58, 491 -499.

[25] Vaubourg, A., 2006, "Differentiation and Discrimination in a Duopoly with Two Bundles," *International Journal of Industrial Organization*, 24, 753-762.

[26] Whinston, M., 1990, "Tying, Foreclosure, and Exclusion," *American Economic Review*, 80, 837-859.

7 Appendix

1. **Proof of Proposition 1**

Let x_1^0 and x_2^0 be the consumer who is indifferent between a bundle and (B_1, A_2) and the consumer indifferent between a bundle and (A_1, B_2), respectively. Then, $x_1^0 = \frac{1}{2} + \frac{p_{1B}^s - p_{1A}^s + \lambda_s}{2t_1}$, and $x_2^0 = \frac{1}{2} + \frac{p_{2B}^s - p_{2A}^s + \lambda_s}{2t_2}$, where $\lambda_s = p_{1A}^s + p_{2A}^s - p_b^s \geq 0$ is the total bundle discount offered by the allied firms. Similarly, there exists a consumer $x_1^1 = \frac{1}{2} + \frac{p_{1B}^s - p_{1A}^s}{2t_1}$ who is indifferent between (A_1, B_2) and (B_1, B_2), and a consumer $x_2^1 = \frac{1}{2} + \frac{p_{2B}^s - p_{2A}^s}{2t_2}$ indifferent between (B_1, A_2) and (B_1, B_2). From Figure 1, the market demands for the allied firm's bundle and stand-alone products are

$$\begin{aligned} D_{bundle}^s &= N\left\{x_1^0 x_2^0 - \frac{\lambda_s^2}{8t_1t_2}\right\}, \\ D_{1A}^s &= N x_1^1(1 - x_2^0), \text{ and} \\ D_{2A}^s &= N x_2^1(1 - x_1^0), \end{aligned}$$

respectively, where $\lambda_s = \sum(1 - \delta_i)p_{iA}^s$. Then firm A_i's profit is written as

$$\pi_{iA}^s = (p_{iA}^s - c_i)D_{iA}^s + (\delta_i p_{iA}^s - c_i)D_{bundle}^s.$$

Using $x_i^0 - x_i^1 = \frac{\lambda_s}{2t_i}$, the first-order condition w.r.t. p_{iA}^s and δ_i can be written as follows.

$$\begin{aligned} \frac{\partial \pi_{iA}^s}{\partial p_{iA}^s} &= x_i^1(1 - x_{-i}^0) + (p_{iA}^s - c_i)\left[-\frac{(1-\delta_i)x_i^1}{2t_{-i}} - \frac{(1 - x_{-i}^0)}{2t_i}\right] \\ &+ (\delta_i p_{iA}^s - c_i)\left[\frac{x_i^1}{2t_{-i}}\right] + \delta_i \underbrace{\left\{(x_1^0 x_2^0 - \frac{\lambda_s^2}{8t_1t_2}) - (\delta_i p_{iA}^s - c_i)\left[\frac{x_{-i}^0}{2t_i} + \frac{x_i^1}{2t_{-i}}\right]\right\}}_{A} = 0 \quad (10) \end{aligned}$$

$$\frac{\partial \pi_{iA}^s}{\partial \delta_i} = \frac{(p_{iA}^s - c_i)x_i^1}{2t_{-i}} + \underbrace{(x_1^0 x_2^0 - \frac{\lambda_s^2}{8t_1t_2}) - (\delta_i p_{iA}^s - c_i)\left[\frac{x_{-i}^0}{2t_i} + \frac{x_i^1}{2t_{-i}}\right]}_{A}. \quad (11)$$

At $\delta_i = 0$, $\frac{\partial \pi_{iA}^s}{\partial \delta_i} = \frac{(p_{iA}^s - c_i)x_i^1}{2t_{-i}} + (x_1^0 x_2^0 - \frac{\lambda_s^2}{8t_1t_2}) + \frac{x_{-i}^0}{2t_i} + \frac{x_i^1}{2t_{-i}} > 0$. Hence, the optimal $\delta_i > 0$. To show that $\left.\frac{\partial \pi_{iA}^s}{\partial \delta_i}\right|_{\delta_i=1} \geq 0$, first we rewrite the first order condition for δ_i using the condition that from (10), the term in the last bracket $A = \frac{1}{\delta_i}$

$\left\{-x_i^1(1-x_{-i}^0)+(p_{iA}^s-c_i)\left[\frac{(1-\delta_i)x_i^1}{2t_{-i}}+\frac{(1-x_{-i}^0)}{2t_i}\right]-(\delta_i p_{iA}^s-c_i)\left[\frac{x_i^1}{2t_{-i}}\right]\right\}$. Plugging this into (11), we get

$$\begin{aligned}\frac{\partial \pi_{iA}^s}{\partial \delta_i} &= \frac{(p_{iA}^s-c_i)x_i^1}{2t_{-i}} \\ &\quad +\frac{1}{\delta_i}\left\{-x_i^1(1-x_{-i}^0)+(p_{iA}^s-c_i)\left[\frac{(1-\delta_i)x_i^1}{2t_{-i}}+\frac{(1-x_{-i}^0)}{2t_i}\right]-(\delta_i p_{iA}^s-c_i)\left[\frac{x_i^1}{2t_{-i}}\right]\right\}.\end{aligned}$$

Then, $\left.\frac{\partial \pi_{iA}^s}{\partial \delta_i}\right|_{\delta_i=1}=-x_i^1(1-x_{-i}^0)+(p_{iA}^s-c_i)\left[\frac{(1-x_{-i}^0)}{2t_i}\right]=(1-x_{-i}^0)(\frac{p_{iA}^s-c_i}{2t_i}-x_i^1)$. Thus, $\left.\frac{\partial \pi_{iA}^s}{\partial \delta_i}\right|_{\delta_i=1}\geq 0$ iff $\frac{p_{iA}^s-c_i}{2t_i}-x_i^1\geq 0$.

From (10), when $\delta_i=1$,

$$\begin{aligned}\left.\frac{\partial \pi_{iA}^s}{\partial p_{iA}^s}\right|_{\delta_i=1} &= x_i^1(1-x_{-i}^0)+(p_{iA}^s-c_i)\left[-\frac{1}{2t_i}\right]+(x_i^0x_{-i}^0-\frac{\lambda_s^2}{8t_1t_2})=0 \\ &\Leftrightarrow \frac{(p_{iA}^s-c_i)}{2t_i}=x_i^1(1-x_{-i}^0)+x_i^0x_{-i}^0-\frac{(x_i^0-x_i^1)(x_{-i}^0-x_{-i}^1)}{2} \qquad (12) \\ &\Leftrightarrow \frac{(p_{iA}^s-c_i)}{2t_i}-x_i^1=(x_i^0-x_i^1)\left(\frac{x_{-i}^0+x_{-i}^1}{2}\right)\geq 0, \qquad (13)\end{aligned}$$

since $x_i^0-x_i^1=\frac{\lambda_s}{2t_i}\geq 0$. Thus, at the optimum $\delta_i=1$, $i=1,2$.

2. **Proof of Proposition 2**

Similarly to the analysis for strategic alliance, we can define y_1^0 and y_2^0 as the consumer who is indifferent between a bundle and (B_1,A_2) and the consumer indifferent between a bundle and (A_1,B_2), respectively. Then, $y_1^0=\frac{1}{2}+\frac{p_{1B}^m-p_{1A}^m+\lambda_m}{2t_1}$, $y_2^0=\frac{1}{2}+\frac{p_{2B}^m-p_{2A}^m+\lambda_m}{2t_2}$, $y_1^1=\frac{1}{2}+\frac{p_{1B}^m-p_{1A}^m}{2t_1}$, and $y_2^1=\frac{1}{2}+\frac{p_{2B}^m-p_{2A}^m}{2t_2}$, where $\lambda_m=p_{1A}^m+p_{2A}^m-p_b^m\geq 0$. The market demands for the merged firm's bundle and stand-alone products are $D_{bundle}^m=N\left\{x_1^0x_2^0-\frac{\lambda^2}{8t_1t_2}\right\}$, $D_{1A}^m=Nx_1^1(1-x_2^0)$, and $D_{2A}^m=Nx_2^1(1-x_1^0)$, respectively. For simplicity, assume that $c_1=c_2=f_1=f_2=0$ and $N=1$ for this proof.[14] The merged firm's profit is

$$\pi_M^m=p_{1A}^m\Gamma_1+p_{2A}^m\Gamma_2-\lambda\Gamma_3,$$

where $\Gamma_1\equiv x_1^1+\frac{\lambda}{2t_1}(x_2^0)-\frac{\lambda^2}{8t_1t_2}$, $\Gamma_2\equiv x_2^1+\frac{\lambda}{2t_2}(x_1^0)-\frac{\lambda^2}{8t_1t_2}$, and $\Gamma_3\equiv x_1^0x_2^0-\frac{\lambda^2}{8t_1t_2}$. Similarly, firm B_i's profits are defined as

$$\pi_{iB}^m=p_{iB}\left\{1-\Gamma_i\right\},\ i=1,2.$$

[14] The market size N, and the fixed costs f_i matter only for the condition under which bundling induces foreclosure.

The proof consists of the following four steps: (1) When the merged firm offers mixed bundling, $\lambda_m > 0$. (2) When the merged firm offers a discount for bundles (i.e., $\lambda_m > 0$), the optimal profit is highest when $t_1 = t_2$. As t_2 grows, the profit decreases. (3) When $t_1 = t_2$, the merged firm earns t_1 by not bundling. The merged firm's profit at the optimal $\lambda_m^* > 0$ is lower than t_1. (4) When $t_2 > t_1$, the merged firm's profit from not bundling is larger than t_1. Hence, in this two-stage game, the merged firm has no incentive to offer mixed bundling as it incurs losses from mixed bundling for all $t_2 \geq t_1$.

(1) If the merged firm offers mixed bundling, $\lambda_m > 0$.

From the first order conditions, firms' best response functions are implicitly defined from the following conditions.

$$\Gamma_1 + \frac{\lambda_m}{2t_1}\left(x_2^0\right) - \frac{p_{1A}^m}{2t_1} - \frac{p_{2A}^m \lambda_m}{4t_1 t_2} = 0 \tag{14}$$

$$\Gamma_2 + \frac{\lambda_m}{2t_2}\left(x_1^0\right) - \frac{p_{2A}^m}{2t_2} - \frac{p_{1A}^m \lambda_m}{4t_1 t_2} = 0 \tag{15}$$

$$\frac{(p_{1A}^m - \lambda_m)}{2t_1}\left(x_2^0\right) + \frac{(p_{2A}^m - \lambda_m)}{2t_2}\left(x_1^0\right) + \frac{\lambda_m^2}{4t_1 t_2} - \Gamma_3 = 0 \tag{16}$$

$$1 - \Gamma_1 = \frac{p_{1B}}{2t_1} \tag{17}$$

$$1 - \Gamma_2 = \frac{p_{2B}}{2t_2}. \tag{18}$$

Combining the two equations (14)and (17) and the two equations (15) and (18),we get

$$2p_{1A}^m \lambda_m = \lambda_m^2 + 12t_1(t_2 - p_{2B}) \tag{19}$$

$$2p_{2A}^m \lambda_m = \lambda_m^2 + 12t_2(t_1 - p_{1B}), \tag{20}$$

respectively. Suppose $\lambda_m = 0$. In this case, $p_{iA}^m = t_i$. Evaluating the first-order condition with respect to λ_m at $\lambda_m = 0$, we obtain $\left.\frac{\partial \pi_M}{\partial \lambda_m}\right|_{\lambda_m = 0} = \frac{1}{4} > 0$. Thus, $\lambda_m > 0$.

(2) For $\lambda > 0$, the merged firm's profits are highest when $t_1 = t_2$.

Combining the equations (14) through (18), we get $\Psi(\lambda, t_1, t_2) = 4\lambda\,(t_1 + t_2) - 3\lambda - 4t_1 t_2 - \frac{64t_1^2 t_2^2 (t_1+t_2)\lambda(-11\lambda^4 + 36t_1t_2\lambda^2 + 216t_1^2t_2^2) + 80t_1^2t_2^2\lambda^2(\lambda^4 + 76t_1t_2\lambda^2 - 432t_1^2t_2^2)}{\left(\lambda^4 - 76t_1t_2\lambda^2 + 144t_1^2t_2^2\right)^2} = 0$. Let $\lambda^*(t_1, t_2) > 0$ be the solution that generates the highest profit π_M^{m*} among all the roots that satisfy $\Psi(\lambda^*, t_1, t_2) = 0$. Assume that such a $\lambda^*(t_1, t_2)$ exists. By construction, $\lambda^*(t_1, t_2)$ is unique. Let $p_{ij}^* \equiv p_{ij}(\lambda^*(t_1, t_2), t_1, t_2)$ be the optimal stand-alone price at $\lambda^*(t_1, t_2)$, for $i = 1, 2, j = A, B$. Plugging the equations (14) through (20) into π_M^m, we obtain

$$\begin{aligned}\pi_M^{m*}(\lambda^*(t_1, t_2), (t_1, t_2)) = \frac{p_{1A}^{m*2}}{2t_1} + \frac{p_{2A}^{m*2}}{2t_2} + \frac{2p_{1A}^{m*} p_{2A}^{m*} \lambda_m^*(t_1, t_2)}{4t_1 t_2} \quad & -6(t_2 - p_{2B}^*) x_2^0(p_{2B}^*, p_{2A}^{m*}, \lambda_m^*) \\ & -6(t_1 - p_{1B}^*) x_1^0(p_{2B}^*, p_{2A}^{m*}, \lambda_m^*).\end{aligned}$$

Let $t_2 = t_1\gamma$, where $\gamma \geq 1$. Then, the profit function $\pi_M^{m*}(\lambda_m^*(t_1,t_2),(t_1,t_2))$ can be rewritten as $\pi_M^{m*}(\lambda_m^*(t_1,\gamma),(t_1,\gamma))$. By Envelope Theorem,[15]

$$\frac{\partial \pi_M^{m*}(\lambda^*(t_1,\gamma),(t_1,\gamma))}{\partial \gamma} = -\frac{p_{2A}^{m*2}}{2t_1\gamma^2} - \frac{2p_{1A}^{m*}p_{2A}^{m*}\lambda_m^*(t_1,t_2)}{4t_1^2\gamma^2} - 6t_1\underbrace{\left(\frac{1}{2}+\frac{p_{2B}^{m*}-p_{2A}^{m*}+\lambda_m^*(t_1,t_2)}{2t_1\gamma}\right)}_{C_1}$$

$$+\frac{6}{2t_1\gamma^2}\underbrace{(t_1\gamma - p_{2B}^*)}_{C_2}\underbrace{(p_{2B}^* - p_{2A}^{m*}+\lambda_m^*(t_1,t_2))}_{C_3}. \tag{21}$$

C_1 is x_2^0, and thus, positive since $0 \leq x_1^0, x_2^0 \leq 1$ by construction. C_2 has to be positive. If not, when $t_2 \leq p_{2B}^*$, combined with (18), the equation (15) reduces to $\frac{3}{2t_2}(t_2 - p_{2B}^*) - \frac{p_{1A}^{m*}\lambda_m^*}{4t_1t_2} - \frac{\lambda_m^{*2}}{8t_1t_2} < 0$, which implies that $t_2 \leq p_{2B}^*$ can't be true at the optimum. If C_3 is negative, $\frac{\partial \pi_M^{m*}(\lambda_m^*(t_1,\gamma),(t_1,\gamma))}{\partial\gamma} < 0$. If C_3 is positive, combining terms in C_1, C_2, and C_3, we get $\frac{\partial \pi_M^{m*}(\lambda_m^*(t_1,\gamma),(t_1,\gamma))}{\partial\gamma} = -\frac{p_{2A}^{m*2}}{2t_1\gamma^2} - \frac{2p_{1A}^{m*}p_{2A}^{m*}\lambda_m^*(t_1,\gamma)}{4t_1^2\gamma^2} - \frac{6}{2t_1\gamma^2}p_{2B}^*\left(p_{2B}^* - p_{2A}^{m*} + \lambda_m^*(t_1,\gamma)\right) - \frac{6t_1^2\gamma}{2t_1\gamma} < 0$. Therefore, $\frac{\partial \pi_M^{m*}(\lambda_m^*(t_1,\gamma),(t_1,\gamma))}{\partial\gamma} < 0$ for all $t_1 > 0$ and $\gamma > 1$, and thus, $\pi_M^{m*}(\lambda_m^*(t_1,t_2),(t_1,t_2))$ is highest when $\gamma = 1$, i.e., $t_1 = t_2$.

(3) When $t_1 = t_2$, $t_1 > \pi_M^{m*}(\lambda_m^*(t_1,1),(t_1,1))$.

When $t_1 = t_2$, if the merged firm does not offer mixed bundling and sells each product separately, $p_{1A} = p_{1B} = t_1$, $p_{2A} = p_{2B} = t_1$, and thus, $\pi_M = \frac{1}{2}(t_1 + t_1) = t_1$. By definition, $\pi_M^{m*}(\lambda_m^*(t_1,1),(t_1,1))$ is the profit when the merged firm offers mixed bundling with a positive discount for a bundle when $t_1 = t_2$. Several papers have shown that mixed bundling is not profitable when $t_1 = t_2$, i.e., $t_1 > \pi_M^{m*}(\lambda_m^*(t_1,1),(t_1,1))$. See Proposition 1 in Matutes and Régibeau (1992) for the proof.

(4) Mixed bundling is never profitable for all $\gamma \geq 1$.

Without bundling, the merged firm earns $\sum_i \pi_{iA}^* = \frac{1}{2}(t_1 + t_1\gamma)$. For any $\gamma \geq 1$, $\sum_i \pi_{iA}^* = \frac{1}{2}(t_1 + t_1\gamma) \geq t_1$. From (3), $t_1 > \pi_M^{m*}(\lambda_m^*(t_1,1),(t_1,1))$. From (2), $\pi_M^{m*}(\lambda_m^*(t_1,1),(t_1,1)) > \pi_M^{m*}(\lambda_m^*(t_1,\gamma),(t_1,\gamma))$ since $\frac{\partial \pi_M^{m*}(\lambda_m^*(t_1,\gamma),(t_1,\gamma))}{\partial\gamma} < 0$ for all t_1 and $\gamma \geq 1$. Combining these results, we get $\sum_i \pi_{iA}^* = \frac{1}{2}(t_1 + t_1\gamma) \geq t_1 > \pi_M^{m*}(\lambda_m^*(t_1,1),(t_1,1)) > \pi_M^{m*}(\lambda_m^*(t_1,\gamma),(t_1,\gamma))$. Therefore, mixed bundling is never profitable for all $\gamma \geq 1$. Q.E.D.

[15] Continuity of $\pi_M^*(\lambda^*(t_1,\gamma),(t_1,\gamma))$ in γ (for $\gamma \geq 1$) only requires that $\lambda^*(t_1,\gamma)$ is continuous in γ. Abusing the notation, we can rewrite $\Phi(\lambda, t_1, t_2)$ as $\Phi(\gamma,\lambda)$. As $\Phi(\lambda,\gamma)$ has continuous derivatives Φ_γ and Φ_λ in a neighborhood of (λ^*,γ) where $\Phi(\lambda^*,\gamma) = 0$, and $\Phi_\lambda(\gamma,\lambda^*) \neq 0$, by Implicit Function theorem, $\lambda^*(t_1,\gamma)$ is continuous in γ. (See *Mathematical Analysis* I [2004, ch 8.5, Proposition 1] by Vladimir Antonovich Zorich for the proof of continuity.)

3. **Proof of Corollary 1.**

Suppose now that B_1 and B_2 are merged to offer mixed bundling, and A_1 and A_2 are considering to offer mixed bundling via strategic alliance. Then the market demands are similar to the ones under a unilateral strategic alliance, and the only change is that $x_1^1 = \frac{1}{2} + \frac{p_{1B}-p_{1A}-\lambda_{mB}}{2t_1}$ and $x_2^1 = \frac{1}{2} + \frac{p_{2B}-p_{2A}-\lambda_{mB}}{2t_2}$ are now a function of λ_{mB}, a bundle discount offered by the merged firm. The proof of Proposition 1 does not depend on whether x_i^1 term has the discount offered by the rival firms or not since the proof does not depend on rival firms' behavior at all. Hence, we get the same result, that allied firms do not want to offer a discount for bundled products even if the rivals are merged and offer a bundle discount under mixed bundling. Then, without a bundle discount, the strategic alliance is equivalent to no bundling. Proposition 2 shows that when rivals are not merged nor offer bundling, it is unprofitable to merge to offer mixed bundling. Thus, when a pair of firms are merged and the other pair of firms form an alliance, mixed bundling is unprofitable. Q.E.D.

4. **Proof of Lemma 1**

Suppose $\alpha^s + \beta > 1$, i.e., $p_{1B}^{sp} + p_{2B}^{sp} - p_{AA} > t_2 - t_1$. Then, $\alpha^s + \beta - \gamma > 0$ since $\gamma < 1$. For a positive market share for B_i, it must be that $\alpha^s + \beta - \gamma < 1$, i.e., $p_{1B}^{sp} + p_{2B}^{sp} - p_{AA} < t_2 + t_1$. Otherwise, $D_{iB}^{sp}(p_{AA}, p_{1B}^{sp}, p_{2B}^{sp}) = 0$ and $\pi_{iB}^{sp} < 0$, and thus, B_i can always do better by slightly lowering its price to restore $D_{iB}^{sp}(p_{AA}, p_{1B}^{sp}, p_{2B}^{sp}) > 0$. Therefore, when $\alpha^s + \beta > 1$, if equilibrium exists, the prices must satisfy $0 < t_2 - t_1 < p_{1B}^{sp} + p_{2B}^{sp} - p_{AA} < t_1 + t_2$. Let s_{iB} be the market share of B_i in market i, for $i = 1, 2$. Note that $s_{iB} < \frac{1}{2}$ in this price range since $\gamma < 1$. The demand for good i from B_i is $D_{iB}^{sp}(p_{AA}, p_{1B}^{sp}, p_{2B}^{sp}) = N s_{iB}$, where $s_{iB} = \frac{\left[t_1+t_2-(p_{1B}^{sp}+p_{2B}^{sp}-p_{AA})\right]^2}{8t_1t_2}$. The demand for a bundle is $D_{AA}(p_{AA}, p_{1B}^{sp}, p_{2B}^{sp}) = D_{1A}^{sp} = D_{2A}^{sp} = N\{1 - s_{iB}\}$. If an equilibrium exists, then from the first-order conditions, we get $p_{1B}^{sp} + p_{2B}^{sp} - p_{AA} = \frac{8t_1t_2[2s_{iB}-1]}{t_1+t_2-(p_{1B}^{sp}+p_{2B}^{sp}-p_{AA})}$. This is a contradiction since $p_{1B}^{sp} + p_{2B}^{sp} - p_{AA} > 0$ and $s_{iB} < \frac{1}{2}$. Therefore, there is no equilibrium in this case. Similarly, there is no equilibrium when $\alpha^s + \beta < \gamma$.Q.E.D.

5. **Proof of Proposition 4**

(1) First, we show that pure bundling is unprofitable if $\alpha^M + \beta > 1$.

Suppose $\alpha^M + \beta > 1$. In equilibrium, it must be that $0 < \alpha^M + \beta - \gamma < 1$ to guarantee $D_{iB}^p > 0$. That is, $t_2 - t_1 < p_{1B}^p + p_{2B}^p - p_M < t_1 + t_2$. The demands for a stand-alone product from B_i and a bundle are $D_{iB}^p(p_M, p_{1B}^p, p_{2B}^p) = N s_{iB}$ and $D_M^p(p_M, p_{1B}^p, p_{2B}^p) = N\{1 - s_{iB}\}$, respectively, where $s_{iB} := \frac{\left\{t_1+t_2-(p_{1B}^p+p_{2B}^p-p_M)\right\}^2}{8t_1t_2}$.

Define $d := p_{1B}^p + p_{2B}^p - p_M$ and $Y := (t_1 + t_2 - d)$. Then, the first-order conditions are given by

$$
\begin{aligned}
p_{iB} - c_i &= \frac{1}{2}Y, \text{ and} && (22) \\
p_M - (c_1 + c_2) &= 2Y - (t_1 + t_2) = \frac{8t_1t_2 - Y^2}{2Y}. && (23)
\end{aligned}
$$

Solving for Y, we get two roots. But in the range where $t_2 - t_1 < d < t_1 + t_2$, $0 < Y < 2t_1$. Since $Y > 0$, the unique solution satisfies

$$\begin{aligned} Y &= t_1 + t_2 - d = \frac{1}{5}\left\{(t_1+t_2) + \sqrt{(t_1+t_2)^2 + 40t_1t_2}\right\}, \text{ or} \qquad (24) \\ d &= (p_{1B}^p + p_{2B}^p - p_M) = \frac{\left\{4(t_1+t_2) - \sqrt{(t_1+t_2)^2 + 40t_1t_2}\right\}}{5} \end{aligned}$$

For the prices that satisfy (24), the condition that $d > t_2 - t_1 \Leftrightarrow 4(t_1 + t_2) - \sqrt{(t_1+t_2)^2 + 40t_1t_2} > 5(t_2 - t_1)$ holds only if $t_1 > \frac{3}{4}t_2$. Thus, when $\alpha^M + \beta > 1$, equilibrium exists only if $t_1 > \frac{3}{4}t_2$. Given that $t_2 > t_1 > \frac{3}{4}t_2$, we can also derive the following conditions for t_1 and t_2.

(i) From (23), for a positive market share $1 - s_{iB} = \frac{8t_1t_2 - Y^2}{8t_1t_2}$, $8t_1t_2 - Y^2 > 0$, and thus, we get $Y > \frac{(t_1+t_2)}{2} \Leftrightarrow (t_1+t_2)^2 < 32t_1t_2$.

(ii) From (i) and (24), $Y < \frac{6(t_1+t_2)}{7}$.

Using these conditions, we can show that bundling is unprofitable in this case. If the merged firm offers pure bundling, in the subgame, with the pricing strategy that satisfies $\alpha^M + \beta > 1$, the merged firm earns $\pi_M^p = (1 - s_{iB})N(p_M - (c_1 + c_2)) - (f_1 + f_2)$. Using the definition of $1 - s_{iB}$ and the conditions in (23), we can rewrite the merged firm's profit function as $\pi_M^p = \frac{2Y}{8t_1t_2}(2Y - (t_1+t_2))^2 N - (f_1 + f_2)$. On the other hand, without bundling, the merged firm earns $\pi_M^p = \frac{N}{2}(t_1 + t_2) - \sum f_i$. Pure bundling by the merged firm is unprofitable if

$$\begin{aligned} &\frac{2Y}{8t_1t_2}(2Y - (t_1+t_2))^2 < \frac{1}{2}(t_1+t_2) \\ \Leftrightarrow\ &(t_1+t_2)(t_1^2 + t_2^2 - 568t_1t_2) + (t_1^2 + t_2^2 + 162t_1t_2)\sqrt{R} < 0, \qquad (25) \end{aligned}$$

where $R = t_1^2 + t_2^2 + 42t_1t_2$.

From (i), the first term in (25) is negative. From (ii), we get $\sqrt{R} < \frac{23}{7}(t_1 + t_2)$. Then, $(t_1+t_2)(t_1^2+t_2^2-568t_1t_2) + (t_1^2+t_2^2+162t_1t_2)\sqrt{R} < (t_1+t_2)(t_1^2+t_2^2-568t_1t_2) + (t_1^2 + t_2^2 + 162t_1t_2)\frac{23}{7}(t_1 + t_2) = \frac{10(t_1+t_2)}{7}(3t_1^2 - 25t_1t_2 + 3t_2^2) < 0$ in the range where $t_2 > t_1 > \frac{3}{4}t_2$. Therefore, pure bundling is unprofitable when $\alpha^M + \beta > 1$.

(2) Suppose $\alpha^M + \beta < 1$.

It must be that $0 < \alpha^M + \beta < 1$ to guarantee $D_M > 0$. That is, $0 < d < t_2 - t_1$. The optimal prices are

$$\begin{aligned} p_M &= \frac{5}{4}t_2 + c_1 + c_2 < p_{1B}^p + p_{2B}^p, \\ p_{iB}^p &= \frac{3}{4}t_2 + c_i, \text{ for } i = 1, 2. \end{aligned}$$

These prices satisfy the condition $d < t_2 - t_1$ only if $t_1 < \frac{3}{4}t_2$. Thus, the outcome is sustained only if $t_1 < \frac{3}{4}t_2$. The merged firm earns $\pi_M^p = \frac{25N}{32}t_2 - \sum f_i$ from pure bundling. Pure bundling is profitable if $t_1 < \frac{9}{16}t_2$. Q.E.D.

6. **Proof of Proposition 6**

If A_1 and A_2 merge, in the subgame nine cases arise: (A-1) when both M and allied rivals offer mixed bundling; (A-2) when M alone offers mixed bundling; (A-3) when allied rivals alone offer mixed bundling; (B-1) when both M and allied rivals offer pure bundling; (B-2) when M offers mixed bundling while allied rivals offer pure bundling; (B-3) when M offers pure bundling while allied rivals offer mixed bundling; (B-4) when M alone offers pure bundling; (B-5) when allied rivals alone offer pure bundling; (C) when no firm offers bundling.

The final outcomes of case (C) are characterized in (1) and (2). The results from cases (A-1) through (A-3) are the same from Propositions 1 and 2 and Corollary 1. If one pair of firms offer pure bundling, the market outcomes are the same regardless of whether the other pair of firms offer mixed bundling, pure bundling, or no bundling. Thus, the results from cases (B-1) through (B-5) are the same.

If A_1 and A_2 do not merge, in the sub-game there are nine cases: (a-1) when both pairs of allied firms offer mixed bundling; (a-2) and (a-3) when one allied firms offer mixed bundling and the other firms do not; (b-1) when both pairs of allied firms offer pure bundling; (b-2) and (b-3) when one allied firms offer mixed bundling while the other allied firms offer pure bundling; (b-4) and (b-5) when only one pair of allied firms offer pure bundling; and (c) when no firms offer bundling. Cases (a-1) through (a-3) are equivalent. As discussed in Section 3.2, when at least one ally offers pure bundling, the outcomes are the same regardless of whether the bundling is unilateral or bilateral. Thus, the results from cases (b-1) through (b-5) are the same.

Summarizing the payoffs from all the cases, we can complete the payoff matrices in two subgames, namely, when A_1 and A_2 merge and when A_1 and A_2 do not merge. The payoff matrices are given in Figure 3. In each subgame illustrated in Figure 3, there are multiple equilibria. However, all equilibria give identical payoffs for the firms. The equilibrium follows from backward induction.

(1) From Proposition **5,** if $\frac{t_1 t_2 N}{(f_1+f_2)} < \frac{16}{9} t_1 < t_2 < \frac{32}{9N} f_2$, merger leads to foreclosure. Thus, A_1 and A_2 merge and monopolize the two markets in this case. Figure 3 does not describe explicit payoffs for merged firm after foreclosure. (See the supplementary Appendix for the full description of payoff after foreclosure) Yet, under the premise that foreclosure leads to a high enough monopoly profit to deter entry (which is always satisfied in the present model), one can easily deduce an outcome where A_1 and A_2 merge and bundle.

(2) In other ranges of parameters, either merger is unprofitable or profitable merger does not induce foreclosure. Thus, firms do not merge, but offer pure bundling via strategic alliances. Q.E.D.

7. **Proof of Lemma 2**

Suppose both pairs of firms (A_1,A_2) and (B_1,B_2) merge.

(1) If both merged firms offer mixed bundling, no merged firm gains. See Armstrong (2006, pp. 123-124) for the proof.

(2) When both merged firms offer pure bundling, let p_{MA} and p_{MB} be the market prices of bundled products offered by the two merged firms, respectively. A consumer with x_{12} buys both goods from the merged firm of A_1 and A_2 if and only if

$$\begin{aligned} & p_{MA} + t_1(x_1)^2 + t_2(x_2)^2 \leq p_{MB} + t_1(x_1-1)^2 + t_2(x_2-1)^2 \\ \Leftrightarrow \quad & x_2 \leq \alpha_{MM} + \beta - \gamma x_1, \end{aligned} \tag{26}$$

where $\alpha_{MM} := \frac{p_{MB}-p_{MA}}{2t_2}$. Solving the first-order conditions, we get $D_{MA} = \frac{1}{2}N$ and $D_{MB} = \frac{1}{2}N$. The optimal prices are

$$\begin{aligned} p_{MA}^* &= t_2 + c_1 + c_2, \\ p_{MB}^* &= t_2 + c_1 + c_2. \end{aligned} \tag{27}$$

The profits are

$$\begin{aligned} \pi_{MA}^* &= \pi_{MB}^* = \frac{1}{2}t_2 N - (f_1 + f_2) \\ &< \frac{1}{2}(t_1 + t_2)N - (f_1 + f_2) = \sum \pi_{iA}^* = \sum \pi_{iB}^*. \end{aligned} \tag{28}$$

(3) When both pairs of firms are merged, but if only one merged firm offers pure bundling. Let p_{MA} and $p_{MB} = p_{1B}^p + p_{2B}^p$ be the market prices of a bundle and two separate products offered by the two merged firms, respectively. A consumer with x_{12} buys both goods from the merged firm of A_1 and A_2 if and only if

$$\begin{aligned} & p_{MA} + t_1(x_1)^2 + t_2(x_2)^2 \leq p_{MB} + t_1(x_1-1)^2 + t_2(x_2-1)^2 \\ \Leftrightarrow \quad & x_2 \leq \alpha_{MM} + \beta - \gamma x_1, \end{aligned} \tag{29}$$

where $\alpha_{MM} := \frac{p_{MB}-p_{MA}}{2t_2}$. Hence, this case is identical to the case (2) when both merged firms offer pure bundling and both merged firms incur losses from bundling. Q.E.D.

8. **Proof of Proposition 7**

Let π_{MA}^m be the post-merger profit for $A_1$and A_2 when both merged firms offer mixed bundling. Also define $\sum \pi_{iA}^m$ as the post-merger profit when the other merged firm alone offers mixed bundling. Similalry, we can define π_{MB}^m and $\sum \pi_{iB}^m$. Figure 4 shows the complete description of payoffs in the subgame when both firms are merged.

(i) From Lemma **2**, Proposition 2, and Armstrong (2006), we get $\sum_i \pi_{ij}^* > \pi_{Mj}^m$ and $\sum_i \pi_{ij}^* > \sum_i \pi_{ij}^m$, $i = 1, 2, j = A, B$.

(ii) Armstrong (2010) shows that if both merged firms offer mixed bundling, in the symmetric equilibrium, the bundle discount λ_m is less than $t_1 = \min\{t_1, t_2\}$. Thus, the bundle price is higher under mixed bundling than under pure bundling. The sum of stand-alone prices under mixed bundling is even higher than the bundle price. On the other hand, from the proof of Lemma **2**, we find that in the symmetric

When both pairs are merged		Merged Firm B		
		Mixed Bundling	Pure bundling	Do not bundle
Merged Firm A	Mixed bundling	π_{MA}^{m} π_{MB}^{m}	π_{MA}^{p} π_{MB}^{p}	π_{M}^{m*} $\sum \pi_{iB}^{m}$
	Pure bundling	π_{MA}^{p} π_{MB}^{p}	π_{MA}^{p} π_{MB}^{p}	π_{MA}^{p} π_{MB}^{p}
	Do not bundle	$\sum \pi_{iA}^{m}$ π_{M}^{m*}	π_{MA}^{p} π_{MB}^{p}	$\sum \pi_{iA}^{*}$ $\sum \pi_{iB}^{*}$

where (1) $\sum_i \pi_{ij}^{*} = \frac{N}{2} \sum_i t_i - \sum_i f_i > \pi_{Mj}^{p} = \frac{N}{2} t_2 - \sum_i f_i$, i=1,2, j=A,B. (2) $\sum_i \pi_{ij}^{*} > \pi_{M}^{m*}$,

(3) $\sum_i \pi_{ij}^{*} > \sum_i \pi_{ij}^{m}$.

Figure 4: Payoffs in the subgame where both pairs of firms are merged.

equilibrium, with pure bundling, each merged firm gets the same market share (the half) as it would under mixed bundling. Hence, with the same market share at higher prices, merged firms earn higher profits under mixed bundling. That is, $\pi_{Mj}^{m} > \pi_{Mj}^{p}$, for $j = A, B$.

(iii) From Matute and Regibeau (1992, Proposition 1) and Armstrong (2006), if the rival firms are also merged and offer mixed bundling, the merged firm earns higher profits by not mixed-bundling, i.e., $\sum_i \pi_{ij}^{m} > \pi_{Mj}^{m}$, $i = 1, 2, j = A, B$.

From (i) through (iii), we find that "not bundle" is a dominant strategy when both firms are merged.

Now we derive the equilibrium of the game in each parameter range.

(1) When $\frac{t_1 t_2 N}{(f_1+f_2)} < \frac{16}{9} t_1 < t_2 < \frac{32}{9N} f_2$.

Eliminating a weakly dominated strategy, we get (not bundle, not bundle) as a unique prediction in the subgame when both pairs of firms are merged. Figure 4 shows the reduced extensive-form game at Stage 0. Then, the unique subgame perfect Nash equilibrium is that both pairs of firms merge, since "merge" is a dominant strategy for both. However, no bundling occurs in equilibrium. (2) When foreclosure is not possible.

The parameter space is divided into two by whether $\frac{16}{9} t_1 < t_2$ or not. Suppose $\frac{16}{9} t_1 < t_2$, i.e., a unilateral merger is profitable. While merger cannot induce foreclosure, in the subgame where only one pair of firms merge, the merged firm always offers pure bundling as it is profitable. But if all the firms merge, firms do not offer

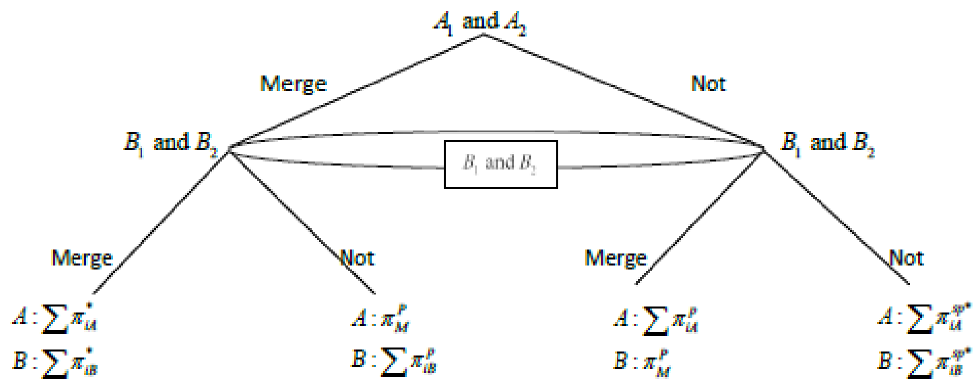

Figure 5: Reduced extensive-form game with a counter merger: when foreclosure is possible.

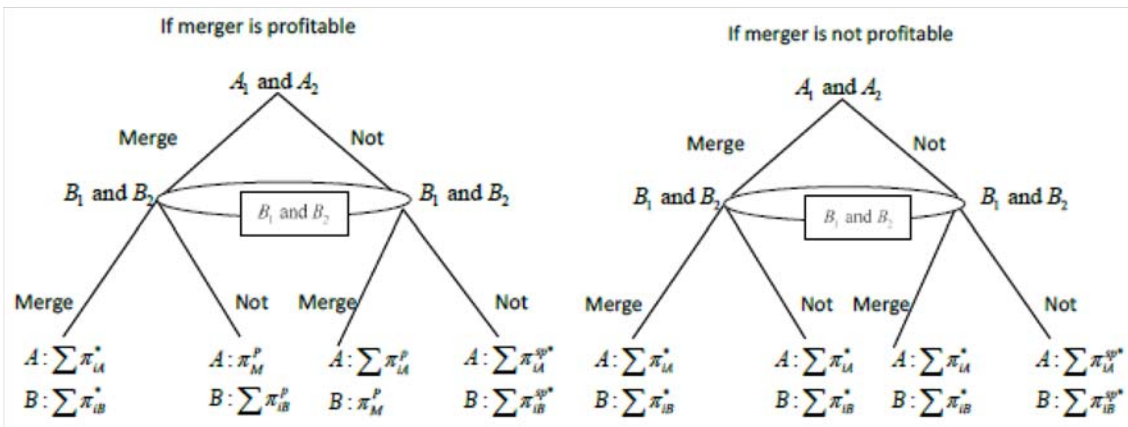

Figure 6: Reduced form extensive games when foreclosure is not possible.

bundling. The payoffs for firms in this case are shown in the first reduced form extensive game in Figure 5. There are two subgame perfect Nash equilibria: (Merge, Merge) and (Not, Not). However, only one of them is an extensive-form trembling hand perfect Nash equilibrium (THPNE).

Suppose $B_1$and B_2 are playing a mixed strategy of choosing to merge with probability ε for $0 < \varepsilon < 1$. Each pair of firms decides not to merge if and only if the expected profits from merging are lower than the expected joint profits from not merging. Then, for a small enough ε, i.e., $\varepsilon < \frac{\sum \pi^{sp*}_{iA} - \pi^p_M}{\sum \pi^{sp*}_{iA} - \pi^p_M + \sum \pi^*_{iA} - \sum \pi^p_{iA}}$, choosing not to merge is a best response for $A_1$and A_2. By symmetry, $B_1$and B_2 also place a minimal weight on "Merge" for a small enough ε when $A_1$and A_2 are playing a mixed strategy of choosing to merge with probability ε. Thus, (Not, Not) is the unique extensive-form trembling hand perfect Nash equilibrium (THPNE).

If $\frac{16}{9}t_1 > t_2$, a unilateral merger is unprofitable. In the subgame in which only one pair of firms merge, "not bundle" is a weakly dominant strategy for all firms. The new payoffs are given in the second extensive-form game in Figure 5. There are two subgame perfect Nash equilibria in this game: (Merge, Merge) and (Not, Not). Only (Not, Not) is the unique extensive-form THPNE of this game. Suppose $B_1$and B_2 are playing a mixed strategy of choosing to merge with probability ε for $0 < \varepsilon < 1$. For all $\varepsilon < 1$, choosing not to merge is a best response for $A_1$and A_2, and the same holds for $B_1$and B_2. Thus, (Not, Not) is the unique extensive-form THPNE. Thus, when foreclosure is not possible, in equilibrium, no firms merge, but firms offer pure bundling through a strategic alliance. Q.E.D.

8 Supplementary Appendix

For a full description of payoffs after foreclosure and successful entry deterrence, consider the following two-period game. If foreclosure does not occur in period 1, then in the second period, firms repeat the game described in section 4, except that in period 2, firms no longer incur fixed costs. If foreclosure occurs, however, the second period game involves new interaction between the merged firm and a potential entrant b_i. Let π_M^F be the monopoly profit after foreclosure. The size of π_M^F depends on the contestability of the markets. Foreclosure is preferable if π_M^F is high enough, which is in general true in the presence of entry barriers. Given the nature of imperfect competition, it is natural to assume that there are entry barriers in the two markets. However, it can be shown that even if $E_i = 0$, foreclosure is preferred to bundling through a strategic alliance.

First of all, by construction, $\overline{p_M^F}$ is always higher than the price at the time of foreclosure, $\widetilde{p_M^*}$. Suppose $E_i = 0$. Since B_2 is making a negative profit at $\widetilde{p_M^*}$, if a potential entrant is making zero profit at $\overline{p_M^F}$, it must be that $\overline{p_M^F} > \widetilde{p_M^*}$. As $\overline{p_M^F} > \widetilde{p_M^*}$ and $D_M^F = N > \widetilde{D_M}$, therefore $\overline{\pi_M^F} > \widetilde{\pi_M^*}$. Thus, even if there is no entry barrier, i.e., $E_i = 0$ for all $i = 1, 2$, the highest profit that deters entry would always be greater than the profit at the time of foreclosure, that is $\overline{\pi_M^F}|_{E_i=0} > \widetilde{\pi_M^*}$. $\overline{p_M^F}$ is higher as E_i increases.

Now we show that profits are higher under foreclosure for any $E_i \geq 0$. If there is no threat of entry, the merged firm sets its bundle price at $v_{12} - t_1 - t_2$, the highest price that induces $D_M^F = N$. Under the possibility of entry, the merged firm charges $\overline{p_M^F}$, the highest price that deters entry. For any given p_M by the merged firm, an entrant $b_i's$ best response function $p_{ib}(p_M)$ satisfies $p_{ib} - c_i = \frac{1}{2} - \widetilde{\alpha} \Leftrightarrow p_{ib} - c_i = \frac{1}{3}(t_2 + p_M - (c_1 + c_2))$, for $i = 1, 2$. An entrant b_i expects a zero profit after a successful entry if $p_M = \overline{p^F_M}$. That is,

$$\begin{aligned}
\pi_{ib}(p_{ib}(\overline{p^F_M})) &= N\frac{(p_{ib}(\overline{p^F_M}) - c_i)^2}{2t_2} - f_i - E_i = 0 \\
&\Leftrightarrow \; p_{ib}(\overline{p^F_M}) - c_i = \sqrt{\frac{(f_i + E_i)2t_2}{N}} \\
&\Rightarrow \; \overline{p^F_M} - (c_1 + c_2) = 3\sqrt{\frac{(f_i + E_i)2t_2}{N}} - t_2.
\end{aligned}$$

Without loss of generality, suppose $(f_1+E_1) < (f_2+E_2)$. Then, at $\overline{p^F_M} \equiv 3\sqrt{\frac{(f_2+E_2)2t_2}{N}} - t_2 + (c_1+c_2)$, $\pi_{1b} > \pi_{2b} = 0$. Entries are deterred in both markets because the merged firm needs to defend only one market to deter entries in both markets. The monopoly profit in period 2 is

$$\begin{aligned}
\overline{\pi^F_M}(E_2; f_2, t_2, N) &\equiv N(\overline{p^F_M} - (c_1 + c_2)) \\
&= N\left(3\sqrt{\frac{(f_2 + E_2)2t_2}{N}} - t_2\right).
\end{aligned}$$

Overall, the merged firm's expected payoff from foreclosure is $\widetilde{\pi^*_M} + \overline{\pi^F_M}$. On the other hand, if A_1 and A_2 do not merge, the firms get $2\sum \pi^{s*}_{iA} + \sum f_i$ from bundling through a strategic alliance. Foreclosure is more profitable than bundling under competition if $\widetilde{\pi^*_M} + \overline{\pi^F_M} > 2\sum \pi^{s*}_{iA} + \sum f_i \;\Leftrightarrow\; (\overline{\pi^F_M} - \sum f_i - \sum \pi^{s*}_{iA}) > \sum \pi^{s*}_{iA} - \widetilde{\pi^*_M} = \frac{7}{32}t_2N \iff E_2 > \frac{5041}{18\,432}t_2N - f_2$. Since $\frac{9}{32}t_2N - f_2 < 0$ when foreclosure is possible, $\frac{5041}{18\,432}t_2N - f_2 < 0$, and thus, the inequality is always satisfied for any $E_i \geq 0$. Q.E.D.