Incentives for Discrimination*

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Abstract

This paper models employers' incentives for discrimination against *ex ante* identical groups of workers when the workers must compete for a limited number of positions. Employers benefit from discrimination against minority workers because it can reduce the overall risk from workers' noisy signals by increasing the expected quality of "majority" workers and their chance to win the competition for the limited number of positions. We show that employers can influence the selection of a discriminatory equilibrium by choosing the set of finalists in competition primarily from a majority group, and favoring them when the two groups are equally qualified. We discuss the implications of equal opportunity laws in this context.

Keywords and Phrases: Statistical Discrimination, Group inequality, Asymmetric information, Cross-Group Risks

JEL Classification Numbers: **D63**, **D82**, **J71**

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1 Introduction

The cause of discrimination has been extensively studied in different disciplines.[1] The statistical discrimination literature, which began with seminal papers by Arrow (1972, 1973), explains that even ex ante identical individuals can be discriminated against as a result of their own self-fulfilling beliefs of discrimination. However, the fundamental question of where the self-fulfilling beliefs originate has remained unanswered.

This paper offers a model in which workers' rational beliefs of discrimination are derived from employers' incentives for discrimination. This model shows that discrimination works as a winner-selection mechanism when workers are in competition for positions. This gives employers incentives to discriminate. Since it is the competition among workers that promotes employers' preference for discrimination toward *ex ante* identical workers, we predict that discrimination is intrinsically prevalent and persistent in any competitive environment even if there are no ex ante differences in the abilities of workers between groups.

This paper models discrimination in an economy in which each firm faces more job candidates than the number of positions it can offer.[2] This structure necessarily creates competition between candidates. In this case, the candidates are not always guaranteed a return on their investment in their human capital. For each candidate, the possibility of receiving a return depends not only on his or her own efforts, but also on whether competing candidates make the same qualifying efforts.[3] Thus, investment incentives are determined by candidates' expected competitive advantages over the other candidates. In this framework, discrimination works as a

[1] The literature on discrimination, a subject first studied formally by Becker (1971) in the field of economics, suggests three broad causes of economic discrimination: discrimination driven by demand-side traits, such as employers' or co-workers' tastes; discrimination driven by supply-side traits, such as different turnover rates for men and women; and statistical discrimination based on self-fulfilling beliefs (see Cain (1986) and England (1992) for examples). Fang and Moro (2011) provides an excellent survey of statistical discrimination literature.

[2] One of the reasons that this structure is common in firms' hiring process is that job seekers typically send out multiple applications. Expecting this, recruiters interview many more job applicants than the number of vacancies.

[3] This nature of competition extends beyond the job market example that we focus on in our modeling. In this context, see the case of the lawsuit against the University of Michigan's law school. (*New York Times*, May 11, 1999).

winner-selection mechanism that encourages the investment of workers from a favored group.

We show that employers benefit from such discrimination. The reason is that, with discrimination, employers can enhance the expected qualification of hired workers when the signals about their ability are noisy. In the case of competition, what matters to employers is only the qualification of *winning* workers. Discrimination increases the investment incentive of the workers in an advantaged group by lowering the incentive of disadvantaged group workers. Thus, if the effect on the advantaged group is significant and their probability of winning increases as a result, discrimination will enhance the overall qualification of winning workers. This indicates *when* discrimination is most likely to occur and, if it does occur, *who* should be the advantaged group. If one group's presence is dominant in the population, workers from the majority group are more likely to be present in any given competition. Then, discrimination in favor of them is likely to enhance their probability of winning and the overall qualification outcomes under competition. Therefore, employers have incentives for discrimination that favors the majority group.[4]

In addition, we discuss possible channels through which employers facilitate discrimination driven by the incentives. We find that employers can actively influence the selection of a discriminatory equilibrium path by choosing a set of competing finalists primarily from a majority group at the pre-selection stage or by implementing an impartial tie-breaking rule. This result underscores the importance of equal opportunity laws. To our knowledge, this paper is the first to model that employers may have incentives for discrimination with *ex ante* identical workers, and show how employers can actively select a path that leads to a discriminatory equilibrium.

In the literature on statistical discrimination, discrimination is understood to be an outcome of intrinsic exogenous differences between groups of workers or of endogenously derived, average differences between groups in equilibrium. In the case of discrimination toward *ex ante* identical groups of workers that we analyze, the discrimination can be explained only in terms of endoge-

[4] Arrow (1973) explains the plausibility of the discriminatory equilibrium with stability and a historical observation of the case with whites and blacks instead. "Thus, discrimination due to deferring performance is possible even though the underlying assumptions are symmetric with respect to race. To discuss the plausibility of this situation, we must look into the stability of alternative equilibria." (p. 29)

nously determined, average group differences that arise as a result of self-fulfilling prophesy.[5] The present paper also builds on a framework of statistical discrimination, and the equilibrium is partially driven by self-fulfilling beliefs. The information structure in this paper resembles that of Coate and Loury (1993). In Coate and Loury (1993), discrimination of *ex ante* identical workers occurs because employers' discriminatory beliefs toward two groups induce such an equilibrium outcome. Since the two groups are treated separately in the model, by changing the employers' beliefs, the disadvantaged group can attain the same high productivity as the advantaged group's, which is a Pareto improvement since it makes the disadvantaged group better off without making anyone else, including the advantaged group and the employers, worse off. Hence, as in Arrow (1972), removing discrimination is, at least theoretically, quite feasible.

Moro and Norman (2004) was the first to question the "convenience" of removing such statistical discrimination. In their general equilibrium model, production technology requires two complementary inputs. When too many workers invest in a high-skills job, the marginal product decreases, generating greater incentives to invest in a low-skills job. Asymmetric equilibrium occurs as a result of two groups' coordination to specialize in different tasks. Hence, removing discrimination always involves a welfare loss by the advantaged group. In a different setting, with a search model, Mailath, Samuelson and Shaked (2000) explains an externality that causes one group's search benefit to depend on the other group's qualification level in showing a conflict of interests between the two groups.

In contrast, our paper focuses on what generates the "incentives" to discriminate against ex ante identical workers in the employers' perspective. Employers prefer such discrimination because it improves the expected qualification of a worker who is hired through competition. As discrimination increases the investment incentives for the workers of a favored group at

[5] In the case of discrimination caused by exogenous differences between groups of workers, the study is focused on the reasons that the groups differ intrinsically. Lang (1986) offers language barriers between employers and minorities as a source of the difference that engenders group inequality. Phelps (1972) and Aigner and Cain (1977) attribute discrimination to the difference in the variance of signals of different groups, although these papers do not explain what causes the different variance among groups. For this aspect, Cornell and Welch (1996) show that discrimination based on different variances can arise in a tournament model in which screening is important and employers who are from one group can more easily screen applicants from the same group than from another group.

the expense of unfavored-group workers, the probability that most of the winners are from the favored group increases. This benefits employers because only the winners' quality matters to the employers. A similar incentive to discriminate symmetric buyers has been analyzed in auction literature. Levin and Smith (1994) and Lu (2009) show that sellers may improve their revenues by limiting the number of potential bidders. As a result, in equilibrium, some of the symmetric bidders are induced to not participate in auction.

This paper also explains why discrimination normally results in an advantage for the majority, but not the converse. Since the increase in benefits from discrimination is greater if the favored group's chance of winning is higher, employers favor a group with a majority standing. Because competition for a position is prevalent in a typical hiring process, the results in our paper imply that discrimination is likely to occur if there are no proper employment policies.

This paper is organized as follows. Section 2 describes the basic model and derives symmetric and asymmetric equilibria. Section 3 shows how discrimination alters the overall risk that each employer faces and derives his preference for discrimination. In Section 4, we discuss the welfare implications of discrimination and the roles of equal opportunity laws. Section 5 contains our conclusion.

2 Model

Consider a market in which there are many identical employers and workers.[6] Workers belong to one of two groups, A and B, and the population's share of group i is $\lambda_i \in (0,1)$, $i = A, B$. Each worker from group i decides whether or not to make an investment in human capital to become qualified. The investment cost c_i of each worker is drawn from a continuous CDF F, which has a density $f > 0$ on a support $[0, \overline{c}]$, $0 < \overline{c}$, for both groups. The group identity is publicly observable at no cost, but each worker's qualification is private information to that worker.[7]

[6] The employers can be managers or admissions officers, and the workers can be job candidates, or college applicants. Hence, the selection decision can be interpreted broadly in the context of admission, as well as employment.

[7] We only consider group characteristics that are not subject to individual choices such as race or sex.

Workers signal their qualification through a test. Signals are noisy in the sense that the signals from a qualified worker may take either H with a probability of $(1-q)$ or M with a probability of q, and the signals from an unqualified worker may be either M with a probability of u or L with $(1-u)$, where $q, u \in (0,1)$.[8] This structure ensures that the distribution of qualified workers' signals first-order stochastically dominates that of unqualified workers' signals. In addition, we assume that $q = \Pr(M|qualified) > u = \Pr(M|unqualified)$. This assumption is required to make workers inclined to invest in qualification even if the signal turns out to be M.[9]

Each employer is randomly matched with two workers from the entire population. After observing workers' test results, the employer selects at most one of them for a position and pays v for the selected worker. Each employer gains a return of $R > 0$ by hiring a qualified worker, and 0 otherwise. We assume that $v \in (0, R)$ and $v < \overline{c}$. Following Coate and Loury (1993) and Blume (2005), v is fixed and group-independent. We call $\rho = v/R$ a wage/output ratio. The payoff for an unselected worker is normalized at 0.

The timing of the game is as follows. At Stage 0, nature chooses the workers' group identity $i = A, B$ and the cost of investment c_i for each group i worker. At Stage 1, workers decide whether or not to invest in human capital. At Stage 2, an employer is randomly matched with two workers and the signals for qualified workers and unqualified workers are determined. At Stage 3, the employer determines who to hire based on the signals. At Stage 4, the quality of the hired worker is revealed and the payoff from hiring is realized.

[8] We use a discrete distribution of signals for simplicity. This does not affect the qualitative results of our paper. Various types of simplified signaling distributions that resemble the present one have been adopted in Blume (2005), Fryer (2007) and Chaudhauri and Sethi (2008). Our distribution is similar to that in Fryer (2007). Such a set-up is reasonable because, even when test scores are continuous, we often classify them for evaluation into discrete measures, such as typical grades at universities and qualifying examinations in doctoral programs.

[9] See Section 2.1. for a detailed explanation of this condition.

2.1 Beliefs

When an employer is randomly matched with two workers from the population, each worker of group A has λ_A chance to compete with a worker of group A and $\lambda_B \equiv (1 - \lambda_A)$ chance to compete with a worker of group B, and vice versa. In the basic model, we assume that each employer has no control over the composition of λ_i in the pool of competing candidates. This assumption will be relaxed in Section 3 when we analyze employers' preference for discrimination.

Let i and j be the two randomly matched workers. The two workers may be from the same group, $i = j$, or from different groups, $i \neq j$. Let α_i (α_j, respectively) be the expected fraction of group i (group j) workers who are qualified with an investment in human capital. Then, the probability that a worker from group i has a signal S_i, $p_S(\alpha_i)$, for $S_i \in \{H, M, L\}$, is derived as follows:

$$\begin{aligned}
p_H(\alpha_i) &\equiv \alpha_i (1 - q), \\
p_M(\alpha_i) &\equiv \alpha_i q + (1 - \alpha_i) u, \\
p_L(\alpha_i) &\equiv (1 - \alpha_i)(1 - u).
\end{aligned}$$

Each worker's strategy is a mapping $Q_i : [0, \bar{c}] \to \{0, 1\}$, where 1 denotes qualified, and $\Pr(Q_i = 1|S_i)$ denotes the conditional probability that a worker i, if chosen, is qualified for a given S_i. From the specified signaling structure, it is clear that, for a given signal H, $\Pr(Q_i = 1|S_i = H) = 1$, and for a given signal L, $\Pr(Q_i = 1|S_i = L) = 0$. For a given signal M, the posterior probability for a worker from group i to be qualified is denoted by $\mu : [0, 1] \to [0, 1]$ and

$$\mu(\alpha_i) \equiv \frac{\alpha_i q}{\alpha_i q + (1 - \alpha_i) u}.$$

Let $\theta = (S_i, S_j) \in \Theta$, $\Theta \equiv \{H, M, L\}^2$, be a vector of observed signals from i and j, i.e., S_i and S_j. For a given θ, the employer's payoff for hiring a worker is

$$\mathbb{E}(u_E|\theta) \equiv \max\{\Pr(Q_i = 1|S_i), \Pr(Q_j = 1|S_j)\} R - v. \tag{1}$$

Each employer's strategy is a mapping $E : \Theta \to \{i, j, 0\}$, where i or j means hiring either i or j, whereas 0 means hiring no one.

The employer strictly prefers a worker with a signal H to one with M, and M to L. To determine the employer's hiring strategy, let us establish that "θ is favorable to i" whenever S_i is strictly preferred to S_j. For example, if $\theta = (H, M)$, θ is favorable to i. With Bayesian inferences based on the observed signals from the two workers, we can determine that it is optimal for the employer to hire i whenever θ is favorable to i and $\mathbb{E}(u_E|\theta) \geq 0$.

If one of the signals S_i is H, $\mathbb{E}(u_E|\theta) > 0$ always. However, when the best available signal is M, the condition $\mathbb{E}(u_E|\theta) \geq 0$ holds if and only if $\mu(\alpha_i) \geq v/R = \rho$. For several cases of such a θ, the condition is written as

$$\frac{\alpha_i q}{\alpha_i q + (1-\alpha_i)u} \geq \rho, \text{ or } \alpha_i \geq \alpha_s \equiv \frac{u\rho}{u\rho + q(1-\rho)}. \tag{2}$$

When $\theta = (M, M)$, if $\alpha_i \geq \alpha_s$, the probability that an employer will choose a worker from group i is a mapping $\varphi : [0,1]^2 \to [0,1]$ with $\varphi(\alpha_i, \alpha_j)$, and a worker j is selected with a probability of $1 - \varphi(\alpha_i, \alpha_j)$ if $\alpha_j \geq \alpha_s$; 0 otherwise. The sequentially rational choice rule is φ^* such that

$$\varphi^*(\alpha_i, \alpha_j) = \begin{cases} 1 & \text{if} \quad \alpha_i > \alpha_j \\ [0,1] & \text{if} \quad \alpha_i = \alpha_j \\ 0 & \text{if} \quad \alpha_i < \alpha_j \end{cases} \tag{3}$$

If the two workers are from the same group i, they will be indistinguishable in their group characteristics and thus systematically differentiating them is not possible. Thus, $\varphi^*(\alpha_i, \alpha_i) = 1/2$.[10]

[10] To the employer, α_i is an "expected" fraction of group i candidates to invest that he derives from the information of the distribution F, because each player's cost is private information. For this reason, strictly speaking, the α_i and α_j that appear in the condition $\varphi(\alpha_i, \alpha_j)$ for employer's decision to hire should be in expected terms, such as α_i^e and α_j^e. However, following the convention in the statistical discrimination literature, we abuse the notation without differentiating them from expected terms. When the nature randomly selects an applicant from each group to match the employer, α_i becomes the employer's belief of the probability that a candidate from group i is qualified.

This reveals that the employer's choice depends not only on how likely it is for each group to be qualified, but also on how likely it is for one group to be more qualified than the other. As a result, each group workers' qualifying effort becomes interdependent in the sense that each group must take into account the other group's qualifying effort in choosing its own. This differentiates our model from other models without a competition.

Now consider each worker's decision to invest. Players in each group make a discrete decision to invest or not to invest. For a given c_i, group i workers invest as long as the expected return on their investment is greater than c_i. Since they differ in their costs of investment, even within a same group and for given same investment incentives, some players will invest, whereas others may not, depending on how costly it is to do so. α_i represents the "fraction" of group i workers who have a sufficiently low cost to invest, given the incentives for signaling. Given that F is the distribution of the investment cost for the workers, each individual decision of group i workers partitions the support of the F distribution into two parts, one with the workers in the range of a low cost who invest, and the other with the workers who don't. α_i is the cumulative density for those workers with a low cost. This is how α_i is determined.

Group i workers' expected return depends on their beliefs about group j workers' qualification α_j and the employer's choice rule φ. Under the assumption of $q > u$, workers have an incentive to invest even if the signal may turn out to be M. Suppose that $\alpha_i, \alpha_j \geq \alpha_s$. When the signal is M, competing against another worker (e.g., a group j worker), a group i worker's chance to be employed is $q[p_M(\alpha_j)\varphi(\alpha_i, \alpha_j) + p_L(\alpha_j)]$ if he makes the investment, whereas the chance is $u[p_M(\alpha_j)\varphi(\alpha_i, \alpha_j) + p_L(\alpha_j)]$ if he does not. Hence, the incentive for investment exists, i.e., $(q-u)[p_M(\alpha_j)\varphi(\alpha_i, \alpha_j) + p_L(\alpha_j)] > 0$, only if $q > u$.

Expecting to compete against another worker from group j ($j = i$ or $j \neq i$), group i workers' incentive to invest in qualification is a function of the net increase in the winning probability, which is summarized by the following function $\widehat{\beta}:[0,1]^2\times[0,1]\rightarrow[0,1]$

$$\begin{aligned}\widehat{\beta}\left(\alpha_i,\alpha_j,\varphi\right) &= \underbrace{(1-q)\left[p_H(\alpha_j)\frac{1}{2}+p_M(\alpha_j)+p_L(\alpha_j)\right]}_{S_i=H} \\ &\qquad +\mathbf{1}_{\{\alpha_i\geq\alpha_s\}}\underbrace{(q-u)[p_M(\alpha_j)\varphi\left(\alpha_i,\alpha_j\right)+p_L(\alpha_j)]}_{S_i=M}\end{aligned} \tag{4}$$

Equation (4) shows that β is a decreasing function of α_j, implying a negative externality of each worker's investment on other workers who compete with them.[11] If $\alpha_i<\alpha_s$, the term for $S_i=M$ disappears and there is no effect of $\varphi\left(\alpha_i,\alpha_j\right)$ in competition with the other worker. However, if $\alpha_i\geq\alpha_s$, $i\neq j$, any difference in investment incentives between the two groups matters (directly) to a worker in group i. A higher $\varphi(\alpha_i,\alpha_j)$ for $i\neq j$ represents group i workers' competitive advantage over group j workers, other things being equal. (3) shows that each of group i workers enjoys the advantage if more of his group workers are investing in human capital than the other group workers, $\alpha_i>\alpha_j$, or merely when the tie-breaking rule is biased in favor of group i, i.e., $\varphi\left(\alpha_i,\alpha_j\right)>1/2$ even if $\alpha_i=\alpha_j$. A group that expect such a competitive advantage makes a higher investment in human capital than otherwise. In Section 3.3, we explain in detail how the expectation of a biased tie-breaking rule can induce asymmetric equilibrium and why the employer might be interested in implementing such a biased rule.

Since a group i worker is matched to another worker from the same group i with a probability of λ_i or a worker from a different group $j\neq i$ with a probability of $1-\lambda_i$, the worker's expected return from investment is

$$\left[\lambda_i\widehat{\beta}\left(\alpha_i,\alpha_i,\varphi\right)+(1-\lambda_i)\widehat{\beta}\left(\alpha_i,\alpha_j,\varphi\right)\right]v. \tag{5}$$

The worker invests as long as $\left[\lambda_i\widehat{\beta}\left(\alpha_i,\alpha_i,\varphi\right)+(1-\lambda_i)\widehat{\beta}\left(\alpha_i,\alpha_j,\varphi\right)\right]v\geq c_i$. Thus, $k_i=[\lambda_i\widehat{\beta}\left(\alpha_i,\alpha_i,\varphi\right)+(1-\lambda_i)\widehat{\beta}\left(\alpha_i,\alpha_j,\varphi\right)]v$ becomes the "cut-off" that matters for group i workers in determining their investment decision. Since $0<[\lambda_i\widehat{\beta}\left(\alpha_i,\alpha_i,\varphi\right)+(1-\lambda_i)\widehat{\beta}\left(\alpha_i,\alpha_j,\varphi\right)]<1$ for

[11] For detailed proof of the externality, see Lemma 1 in the Appendix.

a given $\lambda_i \in [0,1]$ and $q, u \in (0,1)$, the cut-off k_i is always interior, implying that $0 < \alpha_i < 1$.

Let $(\alpha_A^*, \alpha_B^*) \in [0,1]^2$ be an equilibrium fraction of workers in groups A and B investing in human capital. In equilibrium, group i workers' beliefs must be consistent with the actual strategies chosen by group j workers and the employer, so we have $\alpha_i = \alpha_i^*$ and $\varphi(\alpha_i, \alpha_j) = \varphi^*(\alpha_i^*, \alpha_j^*)$. Let $\beta(\alpha_i^*, \alpha_j^*)$ denote $\widehat{\beta}(\alpha_i, \alpha_j, \varphi)$ evaluated at (α_i^*, α_j^*) and $\varphi^*(\alpha_i^*, \alpha_j^*)$. Then, an equilibrium is defined as $(\alpha_A^*, \alpha_B^*) \in [0,1]^2$ such that for each $i \in \{A, B\}$,

$$[\lambda_i \beta(\alpha_i^*, \alpha_i^*) + (1-\lambda_i)\beta(\alpha_i^*, \alpha_j^*)]v = k_i \text{ and } F(k_i) = \alpha_i^*. \tag{6}$$

Define $G_i(k_i, k_j) \equiv \lambda_i \widehat{\beta}(F(k_i), F(k_i), \varphi(F(k_i), F(k_i))) + (1-\lambda_i)\widehat{\beta}(F(k_i), F(k_j), \varphi(F(k_i), F(k_j)))$. Since $F(k)$ is strictly increasing in k, the equilibrium in (6) can be rewritten in terms of $(k_A^*, k_B^*) \in [0, \overline{c}]^2$ so that for each $i \in \{A, B\}$,

$$G_i(k_i^*, k_j^*)v = k_i^*. \tag{7}$$

Similarly, we define k_s as a level at which $\alpha_s = F(k_s)$. In the following two subsections, using (7), we find an equilibrium in terms of k and examine the existence of equilibria defined as (k_A^*, k_B^*) at which $k_A^* = k_B^*$ if symmetric, or $k_A^* \neq k_B^*$, if asymmetric.

2.2 Symmetric Equilibrium

A symmetric equilibrium arises only if $\varphi(\alpha_i, \alpha_j) = 1/2$. A symmetric equilibrium is defined by $k^* \in (0, \overline{c})$ so that

$$G_i(k^*, k^*)v = \beta(F(k^*), F(k^*))v = k^*$$

for $i \in \{A, B\}$. From (4), $\beta(k)$ function differs depending on whether the symmetric k is greater than k_s or not. We define $\beta_h(k) : [0, \overline{c}] \to [0,1]$ and $\beta_l(k) : [0, \overline{c}] \to [0,1]$ as the β function for a symmetric k when $k \geq k_s$ and $k < k_s$, respectively. Then,

$$\beta_h(k) \equiv (1-q)[P_H(k)\frac{1}{2} + P_M(k) + P_L(k)] + (q-u)[P_M(k)\frac{1}{2} + P_L(k)]; \tag{8}$$

$$\beta_l(k) \equiv (1-q)[P_H(k)\frac{1}{2} + P_M(k) + P_L(k)], \tag{9}$$

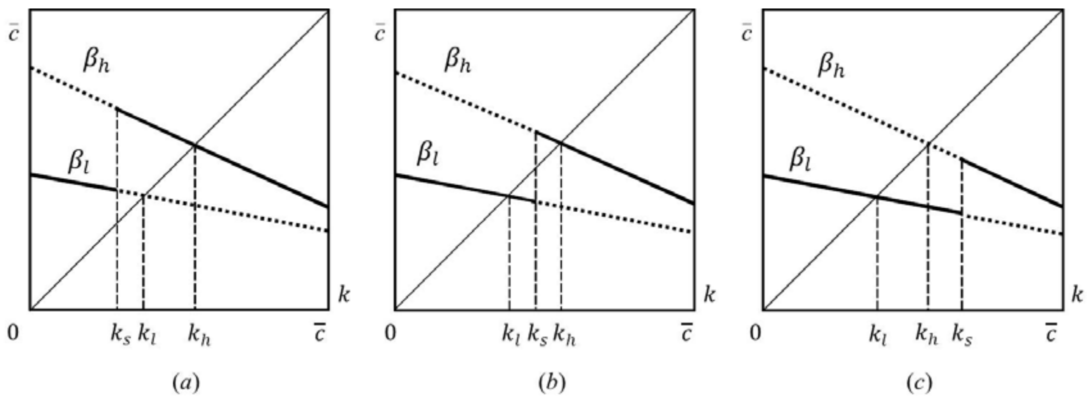

Figure 1: Symmetric equilibria

where $P_S(k)$ is $p_S(\alpha_i)$ defined at $\alpha_i = F(k)$. Note that for any given k, $\beta_h(k) > \beta_l(k)$, and both are continuous in the relevant range. More importantly, each of them is a strictly decreasing function of k, implying that competition reduces a worker's incentive to invest. We can define two fixed points k_l and k_h that satisfy $\beta_l(k_l)\, v = k_l$ and $\beta_h(k_h)\, v = k_h$. $k_l, k_h \in (0, \bar{c})$ are unique and $k_h > k_l$. Figure 1 illustrates the symmetric equilibria when F is uniform. Depending on the level of k_s, which is determined by ρ, there are three possible cases of symmetric equilibria.

Proposition 1 *For a given k_s, three possible cases of symmetric equilibria are characterized as follows.*

(i) If $k_s \le k_l$, there is a unique symmetric equilibrium **HS** *at which $k^* = k_h$.*

(ii) If $k_l < k_s \le k_h$, the equilibrium is either at **LS** *with $k^* = k_l$ or at* **HS** *with $k^* = k_h$.*

(iii) If $k_s > k_h$, there is a unique symmetric equilibrium **LS** *at which $k^* = k_l$.*

k_s represents a minimum group standard that makes hiring a worker of that group worthwhile for the employer when the signal is M. Since k_s is defined at α_s and from (2) α_s depends on ρ, as ρ increases k_s increases. Suppose the wage v is sufficiently low in comparison to the employer's return from selecting a qualified worker, R. A low wage reduces the employer's expected risk of hiring a worker of signal M, and thus increases the employer's willingness to hire. This, in turn, increases the worker's incentive to invest. Hence, in this case, a low $\rho = v/R$ results in a high level of investment (**HS**) by the workers in symmetric equilibrium. Instead, if ρ is sufficiently high, the employer faces a greater risk from hiring a worker with M, and the symmetric equilibrium exhibits a low level of investment. Thus, the equilibrium investment levels are higher when ρ (k_s) is lower.

Suppose that $(1 - \lambda_i) = 0$. The equilibrium in this case is equivalent to the symmetric equilibrium described in the above when $(1 - \lambda_i) \neq 0$. That is, any symmetric equilibrium outcome is equivalent to a case in which a worker from group i never expects to be differentiated by her group identity λ_i. In the next section, we derive equilibrium when workers could be differentiated by the group they belong to and the equilibrium outcome is not symmetric.

2.3 Asymmetric Equilibrium

For the characterization of asymmetric equilibria, consider the case where $k_i > k_j$, without loss of generality. Deriving asymmetric equilibria is similar to deriving symmetric equilibrium. For a detailed configuration of asymmetric equilibria, see the Appendix 5.2. In asymmetric equilibrium, the cut-offs for two groups, k_i and k_j, may be extremely asymmetric, **EA**, or moderately asymmetric, **MA**.

Proposition 2 *Let* **EA** *and* **MA** *be asymmetric equilibrium cut-offs* (k_i^e, k_j^e) *and* (k_i^m, k_j^m) *satisfying* $k_i^e > k_s > k_j^e$ *and* $k_i^m > k_j^m > k_s$*, respectively. Then,*

(i) (k_i^e, k_j^e) *and* (k_i^m, k_j^m) *exist, and*

(ii) $0 < k_j^e < k_j^m < k_i^m < k_i^e < \overline{c}$.

Asymmetric equilibrium exists, either at **EA** or **MA**, as long as k_s is not too high.

Proposition 3 *Let* $\underline{k} \equiv k_j^e$, $\widetilde{k} \equiv k_j^m$*, and* $\overline{k} \equiv k_i^e$*, where* $0 < \underline{k} < \widetilde{k} < \overline{k} < \overline{c}$*. Then,*

(i) For $k_s \leq \underline{k}$ *,* **MA** *is a unique asymmetric equilibrium.*

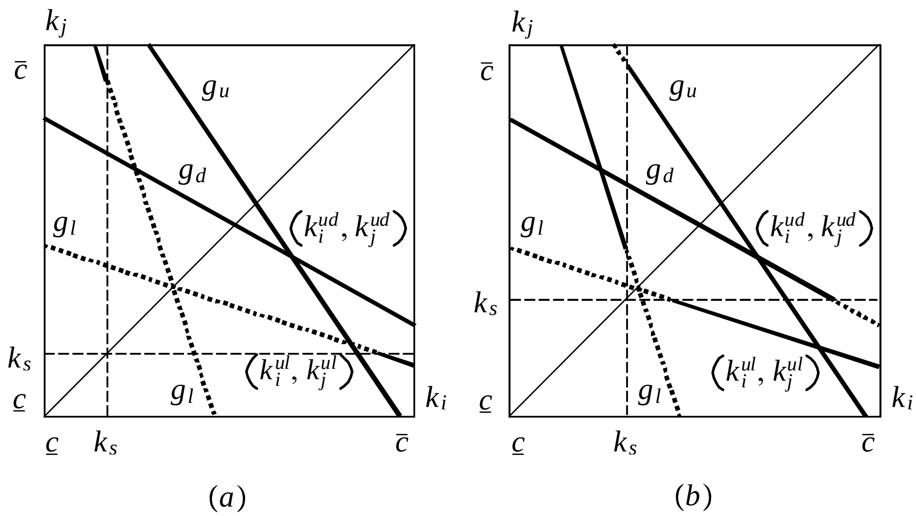

Figure 2: Asymmetric Equilibria

(ii) For $\underline{k} < k_s \leq \widetilde{k}$*, equilibrium occurs either at* **EA** *or at* **MA***.*

(iii) For $\widetilde{k} < k_s \leq \overline{k}$*,* **EA** *is a unique asymmetric equilibrium.*

(iv) For $k_s > \overline{k}$*, there is no asymmetric equilibrium.*

With regard to asymmetric equilibrium, an important difference to note is in how one group gains a competitive advantage in the employer's hiring decision, i.e., $\varphi(\alpha_i, \alpha_j)$. When workers face competition with others who are treated equally, we have $\varphi(\alpha_i, \alpha_j) = 1/2$, and there is no competitive advantage. In contrast, if the group identities $i \neq j$ are factored in, the advantage can be $\varphi(\alpha_i, \alpha_j) = 1$ as long as $\alpha_i > \alpha_j$. This implies that the investment of winning-group workers will be greater under asymmetric equilibrium than under symmetric equilibrium. The winning group's incentive to invest increases further as the disadvantaged group's investment incentive dwindles.

The competitive incentive is higher as the return from winning (ρ) increases. If ρ is so low that k_s is sufficiently low, most workers are willing to invest in human capital, and thus, workers in each group will have a high probability of competing with another qualified worker. Since one group's investment imposes a negative externality on the other group's investment, this implies that workers in an advantaged group i do not see much competitive advantage from investing in qualification given a favorable condition in selection. Thus, when there is a high probability that both groups will invest, the resulting equilibrium exhibits only a moderate difference in their equilibrium investments (**MA**). As the level of ρ grows, workers expect that a disadvantaged group will have a low probability of qualification below the cut-off k_s. Such an expectation enhances an advantaged group's incentive to invest further. Thus, significantly asymmetric investments between two groups can occur in equilibrium (**EA**).

2.4 Synthesis

We can characterize the difference in two groups' incentives to invest and their expected qualification levels between symmetric and asymmetric equilibria in terms of a group $i-$specific dominance. This dominance relationship is useful in explaining the trade-off between discriminatory equilibrium and symmetric equilibrium and the motivation for discrimination in the next section.

Definition 1 *An asymmetric allocation $x = (x_i, x_j)$ i-dominates $y = (y_i, y_j)$, which is denoted by $x >_D y$, if $x_i > y_i$ and $x_j < y_j$, for $x, y \in \mathbb{R}^2$.*

Proposition 4 shows that **MA** i-dominates the symmetric equilibrium **HS**, and **EA** i-dominates the symmetric equilibrium **LS**. Combining Propositions 3 and 4, we find that **EA** i-dominates the symmetric equilibrium **HS**.

Proposition 4 *In equilibrium,* **MA** $>_D$ **HS** *and* **EA** $>_D$ **LS**.

Combining the results from Propositions 1 through 4, we obtain Proposition 5.

Proposition 5 *There are three levels of wage/output ratios $\overline{\rho} = \mu(F(\overline{k}))$, $\rho_h = \mu(F(k_h))$, and $\underline{\rho} = \mu(F(\underline{k}))$, where $\overline{\rho} > \rho_h > \underline{\rho} > 0$, for which*

(i) if $\rho > \overline{\rho}$, the unique equilibrium is **LS**,

(ii) if $\rho_h < \rho \leq \overline{\rho}$, equilibrium occurs at either **EA** *or* **LS***,*

(iii) if $\underline{\rho} < \rho \leq \rho_h$, any one of the four types of equilibria among **EA***,* **MA***,* **HS***, or* **LS** *is possible,*

(iv) if $\rho \leq \underline{\rho}$, equilibrium occurs at either **MA** *or* **HS***.*

In this equilibrium configuration, two factors matter: (I) whether the workers' investment incentives are high enough to warrant the risk of considering a candidate with a signal M for the employer, i.e., whether $k_i > k_s$,and (II) whether a group gains a competitive advantage over the other group, i.e., whether $k_i > k_j$.[12]

In symmetric equilibrium, only (I) matters. In asymmetric equilibrium, an additional benefit of competitive advantage from (II) further enhances the investment incentives of workers from an advantaged group at the expense of the incentives of workers from a disadvantaged group. If the disadvantaged group's investment incentive decreases to a level below k_s from (18), resulting in **EA**, the second effect (II) is much greater than the case in which it remains above k_s and results in **MA**. Naturally, it is ρ that determines the sizes of (I) and (II) through its impact on k_s. Figure 3 (b) and (c) characterize a set of feasible equilibria for this model, whereas the multiple equilibria in Figure 3 (a) describe the case of typical statistical discrimination models.

Proposition 4 shows that a mere differentiability of two groups enhances the incentives for investment in human capital through a competitive advantage for a winning group. This implies that for a given wage, the employer has a higher expected profit from hiring a worker from a group with a competitive advantage. This becomes an important motive for discrimination. In the following sections, our analysis proves this property of asymmetric equilibria.

[12] Following the convention in the statistical discrimination literature, we derive the equilibrium (α_i, α_j) under rational expectations in the sense that all players' beliefs need to be correct and consistent with workers' actual investment decisions. In the framework of statistical discrimination, what determines equilibrium is the players' beliefs. **MA** arises in equilibrium because *all* players expect (α_i, α_j) to be at the level of **MA**. Therefore, this implies that in rational expectations equilibrium, all players have *the same beliefs* about (α_i, α_j). For example, in the range where $\rho \leq \underline{\rho}$, although both **MA** and **HS** are possible, if **MA** arises in equilibrium, it is when *all* the players correctly expect the level of (α_i, α_j) to be at the level of **MA**.

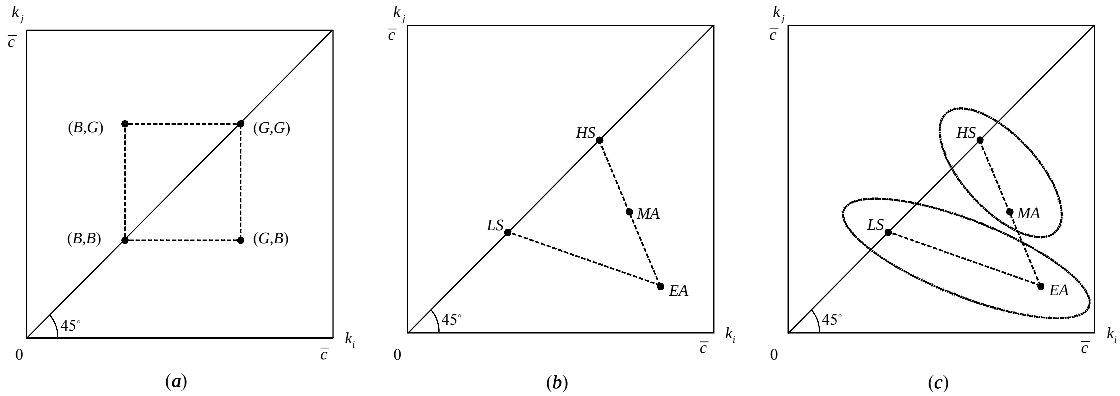

Figure 3: Feasible equilibrium sets

3 Discrimination

In this section, we analyze why employers may prefer discrimination toward *ex ante* identical groups. Comparing an employer's expected payoff from symmetric equilibrium with that from asymmetric equilibrium, we find that discrimination reduces a risk in hiring a worker of a favored group as it lowers the probability that the worker is unqualified. Although it increases the risk of hiring a worker from a disadvantaged group, and thus, creates a group-wise spread of the risks, such a spread can lower the overall risks to the employer as long as the disadvantaged group workers are less frequently considered than the favored group workers. Therefore, in such a case, discrimination promotes the employer's preference for discrimination.

As for the reason that engenders discrimination over *ex ante* identical groups, statistical discrimination literature has been quiet about anything beyond the mechanism of self-fulfilling expectation. The literature explains that, because workers of two groups expect unequal treatment, they make different levels of investment in human capital. Thus, their expectation fulfills a discriminatory outcome, although there is no *ex ante* reason why they should have such an expectation of unequal treatment. In such a process, employers do not play a role in determining the discriminatory outcomes. Instead, employers' decision to discriminate against one group is simply a rational and fair response to workers' asymmetric investment choices, because one group has a higher probability of qualification than the other due to their self-fulfilling expectation of unequal treatment.

In contrast, our paper explains an active role of employers' preference for discrimination in fulfilling the expectation. We consider several channels through which employers can implement their preferred discriminatory outcome.

3.1 Employer's Expected Payoff and Risk

Let $U(\alpha_i, \alpha_j)$ be the expected payoff of the employer who anticipates being matched with two workers drawn from groups i and j. Let $P(H \vee H)$ be the probability that at least one of the two workers has a signal H. Similarly, we can define $P(S_i, S_j)$ as the probability that group i's worker's signal is S_i and group j's worker's signal is S_j. Then,

$$\begin{aligned} U(\alpha_i, \alpha_j) &= P(H \vee H)(R - v) + P(M, M)\Gamma(\alpha_i, \alpha_j) \\ &\quad + \mathbf{1}_{\{\alpha_i \geq \alpha_s\}} P(M, L)[\mu(\alpha_i) R - v] + \mathbf{1}_{\{\alpha_j \geq \alpha_s\}} P(L, M)[\mu(\alpha_j) R - v], \end{aligned} \tag{10}$$

where

$$\Gamma(\alpha_i, \alpha_j) \equiv \begin{cases} \{\mu(\alpha_j) + [\mu(\alpha_i) - \mu(\alpha_j)]\varphi(\alpha_i, \alpha_j)\} R - v & \text{if } \alpha_i \geq \alpha_s \text{ or } \alpha_j \geq \alpha_s, \\ 0 & \text{otherwise.} \end{cases}$$

Denote by $V(\alpha)$ the payoff when both workers have the same α, i.e., $V(\alpha) = U(\alpha, \alpha)$. Then, given that a λ_i fraction of the population belongs to group i, if the two workers are selected in proportion to the size of their representation in the population, the employer's total expected payoff can be described as follows.

$$W(\alpha_i, \alpha_j, \lambda_i) = \underbrace{\lambda_i^2 V(\alpha_i) + (1 - \lambda_i)^2 V(\alpha_j)}_{Intra-group} + \underbrace{2\lambda_i(1 - \lambda_i) U(\alpha_i, \alpha_j)}_{Inter-group}. \tag{11}$$

(11) shows that $W(\alpha_i, \alpha_j, \lambda_i)$ is composed of a couple of different risk components depending

on whether the interactions occur within the same group members (intra-group) or between the members of two different groups (inter-group). In a symmetric equilibrium, an employer expects the same levels of risks, represented by the same levels of α, for all workers, regardless of which group they belong to. The employer's expected payoff is $W(\alpha, \alpha, \lambda_i) = V(\alpha)$. Thus, the symmetric equilibrium removes any potential impact from the only available distinction between the groups, λ_i. Consequently, the risks at symmetric equilibrium are equivalent to the risks of having competition within only one group. For the symmetric equilibrium, it can be shown that the employer's expected payoff is strictly increasing in qualification α.

Proposition 6 $V(\alpha)$ *is a strictly increasing function of* α *on* $[0, 1]$.

Introducing asymmetry between the two groups affects $W(\alpha_i, \alpha_j, \lambda_i)$ in three ways. First, it spreads the risks across the groups, thereby lowering the risk of hiring a worker from an advantaged group below the symmetric equilibrium level while increasing the risk from a disadvantaged group above the level. This implies a different level of risk that the employer faces, depending on whether the group in consideration is favored. Second, the importance of each group's within-group interactions differs. Each group's size of representation in the population matters since a greater weight (λ_i^2) is given to the interactions within the majority group i candidates than to the interaction with the minority group j candidates $((1 - \lambda_i)^2)$. Thus, the employer's $W(\alpha_i, \alpha_j, \lambda_i)$ improves if the majority group's risk is reduced. Third, in the case of inter-group interactions, discrimination is more likely to enhance the expected qualification, as the workers from an advantaged group tend to have a higher qualification and the employer is less likely to select a low-quality worker from the group. In the next subsection, we show how these factors motivate the employer's preference for discrimination.

3.2 Incentive for Discrimination

The employer may prefer discriminatory equilibrium to symmetric equilibrium. One of the main reasons is that each group's size of representation in the population λ_i differs. Suppose that a group has a significantly larger λ_i. Spreading the investment incentives across groups improves the qualification from the majority group. Since the employer expects more frequent interactions with the majority group, the decrease in the majority group's risk can easily outweigh the increase in the minority group risk if λ_i is large enough. In this way, discrimination can lower the overall risks that the employer faces. The following analysis derives the *sufficient* conditions for this result.

To compare an employer's expected payoff from symmetric equilibrium with that from asymmetric equilibrium, we study cases (ii) and (iv) of Proposition 5, where only one type of asymmetric equilibrium is available as an alternative to a symmetric equilibrium. When $\rho \leq \underline{\rho}$, **MA** is the only alternative to **HS**, and when $\rho_h < \rho \leq \overline{\rho}$, **EA** is the only alternative to **LS**. The employer's expected payoff at an equilibrium category $E \in \{\mathbf{HS}, \mathbf{LS}, \mathbf{MA}, \mathbf{EA}\}$ is denoted as

$$W_E(\lambda_i) = W(\alpha_i, \alpha_j, \lambda_i) \text{ where } (\alpha_i, \alpha_j) \text{ is at } E,$$

and note that $W_{HS}(\lambda_i)$ and $W_{LS}(\lambda_i)$ are, in fact, constants, i.e., for any $\lambda_i' \neq \lambda_i$, $W_{HS}(\lambda_i') = W_{HS}(\lambda_i)$, and $W_{LS}(\lambda_i') = W_{LS}(\lambda_i)$.

Suppose $\rho \leq \underline{\rho}$. In this range, there is a trade-off between **MS** and **HS**. The symmetric equilibrium induces $\alpha_h = F(k_h)$, whereas in asymmetric equilibrium, $\alpha_i^m = F(k_i^m) > \alpha_h = F(k_h) > \alpha_j^m = F(k_j^m)$. The effect of moving from a symmetric equilibrium **HS** to an asymmetric equilibrium **MA** on the employer's payoff is

$$\begin{aligned} W_{MA}(\lambda_i) - W_{HS}(\lambda_i) = {} & \lambda_i^2 [V(\alpha_i^m) - V(\alpha_h)] + (1-\lambda_i)^2 [V(\alpha_j^m) - V(\alpha_h)] \\ & + 2\lambda_i(1-\lambda_i)[U(\alpha_i^m, \alpha_j^m) - V(\alpha_h)]. \end{aligned} \tag{12}$$

The first and second terms in (12) refer to the effect created by introducing a spread in the symmetric qualification level α_h within the same group interactions (intra-group effect). The last term shows the effect on the payoff of inter-group interactions (inter-group effect). Since $V(\alpha)$ is strictly increasing in α from Proposition 6, the first term $[V(\alpha_i^m) - V(\alpha_h)]$ is positive, whereas the second term $[V(\alpha_j^m) - V(\alpha_h)]$ is negative. Suppose α_i^m and α_j^m are (implicit) differentiable functions of λ_i. Then, as λ_i converges to 1, (α_i^m, α_j^m) converges to (α_h, α_h), and $W_{MA}(\lambda_i) - W_{HS}(\lambda_i) \to 0$. In addition, if

$$\lim_{\lambda_i \to 1} \frac{d[W_{MA}(\lambda_i) - W_{HS}(\lambda_i)]}{d\lambda_i} < 0, \tag{13}$$

there exists $\lambda_i \in (0,1)$ sufficiently close to 1 such that $W_{MA}(\lambda_i) - W_{HS}(\lambda_i) > 0$. Since (α_i^m, α_j^m) converges to (α_h, α_h) as λ_i converges to 1, the derivative in (13) reduces to

$$\lim_{\lambda_i \to 1} \frac{d[W_{MA}(\lambda_i) - W_{HS}(\lambda_i)]}{d\lambda_i} = V'(\alpha_h) \lim_{\lambda_i \to 1} \frac{d\alpha_i^m}{d\lambda_i}.$$

Thus, the critical conditions are the differentiability of α_i^m and α_j^m from the implicit function theorem and the limit behavior of the derivative of α_i^m with respect to λ_i. Proposition 7 shows that for $\lambda_i \in (0,1)$ sufficiently close to 1, the employer's equilibrium payoff is higher at the discriminatory equilibrium.

Proposition 7 *Suppose $\rho \leq \underline{\rho}$. Then, there exists $\lambda_i \in (0,1)$ sufficiently close to 1 such that $W_{MA}(\lambda_i) > W_{HS}(\lambda_i)$, and therefore, the employer prefers discrimination.*

When $\rho_h < \rho \leq \overline{\rho}$, there is a trade-off between **EA** and **LS**. The symmetric equilibrium induces $\alpha_l = F(k_l)$, whereas in asymmetric equilibrium, $\alpha_i^e = F(k_i^e) > \alpha_l = F(k_l) > \alpha_j^e = F(k_j^e)$, and the effect of discrimination on the employer's payoff is as follows.

$$\begin{aligned} W_{EA}(\lambda_i) - W_{LS}(\lambda_i) = \lambda_i^2 [V(\alpha_i^e) - V(\alpha_l)] + (1-\lambda_i)^2 [V(\alpha_j^e) - V(\alpha_l)] \\ +2\lambda_i(1-\lambda_i)[U(\alpha_i^e, \alpha_j^e) - V(\alpha_l)] \end{aligned} \tag{14}$$

As in (12), the first and second terms in (14) refer to the effect created by introducing a spread in the symmetric qualification level α_l within the same group interactions (intra-group effect). The last term shows the effect on the payoff from inter-group interactions (inter-group effect).

Since

$$\lim_{\lambda_i \to 1} \frac{d[W_{EA}(\lambda_i) - W_{LS}(\lambda_i)]}{d\lambda_i} = V'(\alpha_l) \lim_{\lambda_i \to 1} \frac{d\alpha_i^e}{d\lambda_i},$$

the differentiability of α_i^e and α_j^e and the limit behavior of the derivative of α_i^e with respect to λ_i derive the following result.

Proposition 8 *Suppose $\rho_h < \rho \leq \overline{\rho}$. Then, there exists $\lambda_i \in (0,1)$ sufficiently close to 1 such that $W_{EA}(\lambda_i) > W_{LS}(\lambda_i)$, and therefore, the employer prefers discrimination.*

Propositions 7 and 8 underline the importance of λ_i, the group representation in the labor market, in motivating employers' preference for discrimination. If the size of λ_i is naturally determined, it implies that one group's majority standing in the population naturally advocates the employer's preference to favor the group.

To consider the employer's incentive for discrimination in a broader range of parameters, let us refine the equilibria by considering the stability. The following Proposition 9 shows that **HS** is not stable.

Proposition 9 **HS** *is not stable, whereas* **LS, EA** *and* **MA** *are locally stable.*

When only stable equilibria are concerned, the employers' incentive to discriminate is stronger. Suppose $\rho \leq \overline{\rho}$.[13] In this range, the set of possible stable equilibria is $\{\mathbf{LS}, \mathbf{MA}, \mathbf{EA}\}$. In particular, when $\rho \leq \underline{\rho}$ (case (iv) of Proposition 5), **MA** is the unique stable equilibrium. When $\underline{\rho} < \rho \leq \rho_h$ (case (iii) of Proposition 5), **LS** becomes the only stable symmetric equilibrium, whereas both **EA** and **MA** are available as alternatives to stable symmetric equilibrium **LS**. Since Proposition 8 shows that **EA** is preferred to **LS** for a sufficiently large enough λ_i, this implies that in this range, the employer would prefer discriminatory equilibrium as well.

[13] When $\rho > \overline{\rho}$, a high ρ makes the signal M worthless to the employer, and thus, the employer is not willing to hire any worker unless the signal is H. Consequently, there is not enough incentive for investment, and the only equilibrium occurs at **LS** in this range. Thus, a meaningful signaling occurs only if ρ is not too high, $\rho \leq \overline{\rho}$, so that the noisy signal M becomes valuable to the employer and there is enough investment incentive for the workers.

Corollary 1 *Consider only stable equilibria. Suppose* $\underline{\rho} < \rho \leq \rho_h$*. If* $\lambda_i \in (0,1)$ *is sufficiently close to* 1*,* $W_{EA}(\lambda_i) > W_{LS}(\lambda_i)$*, and therefore, the employer prefers discriminatory equilibrium.*

Until now, we have considered a case in which the employer only responds passively to the signals of competing workers by selecting a worker with a better signal, which is a standard framework of analysis in statistical discrimination literature. Discrimination in such a framework can occur only as a result of workers' self-motivated differential investment incentives in human capital. However, given that workers are ex ante identical, there is no prior reason that the two groups' workers should expect different group returns on their investment. Our paper provides a rationale for employers to seek a discriminatory outcome. Since the employer may benefit from a discriminatory equilibrium, if he has a means of influencing the selection of asymmetric equilibrium over symmetric equilibrium, discrimination is no longer purely self-fulfilling. In the following section, we show how the employer can influence the workers' equilibrium beliefs and the type of equilibrium.

3.3 Influence on the Selection of Discriminatory Equilibrium

Our framework highlights two channels through which employers can influence equilibrium selection when they prefer discriminatory equilibrium. These are (i) pre-selection of the pool of job candidates and (ii) a tie-breaking rule.

Employers may be able to influence the equilibrium by the choice of an "effective" λ_i. Until now, we have assumed that nature selects the size of the two groups λ_i in the hiring process. However, employers do have some degree of control over the selection of the pools of job candidates, especially in the final stage of recruiting, in deciding whether or not the finalists are more likely to be from one group.

For this, consider an extended model where each employer first pre-selects a pool of candidates based on the group characteristics. Suppose that, at Stage 2, if the workers are from group j, the employer pre-approves the group j workers to be included in the next round of the hiring process only with a $\sigma \in (0,1)$ probability. From these pre-selected workers, two workers are randomly matched. In this case, the employer essentially determines what fraction of the finalists will be selected from group i, $\widehat{\lambda}_i$, $i = A, B$.[14] For a group i worker, the intra-group probability of competition and the inter-group probability of competition are

$$\widehat{\lambda}_i = \frac{\lambda_i}{\lambda_i + \sigma(1-\lambda_i)} \text{ and } 1 - \widehat{\lambda}_i = 1 - \frac{\lambda_i}{\lambda_i + \sigma(1-\lambda_i)},$$

whereas for a group j worker, the intra-group probability of competition, the inter-group probability of competition and the probability of being not-selected are $\sigma\widehat{\lambda}_j$, $\sigma(1-\widehat{\lambda}_j)$ and $(1-\sigma)$, respectively, where $\widehat{\lambda}_j \equiv 1 - \widehat{\lambda}_i$. Since $\widehat{\lambda}_i$ is determined before the employer observes workers' signals, incorporating the employer's choice of σ (and thus $\widehat{\lambda}_i$) in our framework does not affect the analysis in the previous sections. To see the effect on workers' incentives to invest in human capital, let $\widehat{k}_i$ be the "cut-off" in (5) for group i workers for a given $\widehat{\lambda}_i$. Then,

$$[\widehat{\lambda}_i\widehat{\beta}(\alpha_i, \alpha_i, 1/2) + (1-\widehat{\lambda}_i)\widehat{\beta}(\alpha_i, \alpha_j, \varphi)]v = \widehat{k}_i.$$

The "cut-off" for group j workers is

$$[\sigma(1-\widehat{\lambda}_i)\widehat{\beta}(\alpha_j, \alpha_j, 1/2) + \sigma\widehat{\lambda}_i\widehat{\beta}(\alpha_i, \alpha_j, \varphi)]v = \widehat{k}_j.$$

Evaluating the incentives at the symmetric levels of investment $\alpha = \alpha_i = \alpha_j$, we obtain

$$\widehat{k}_i = \widehat{\beta}(\alpha, \alpha, 1/2) > \sigma\widehat{\beta}(\alpha, \alpha, 1/2) = \widehat{k}_j.$$

This shows that for $\sigma < 1$, group j workers' expected return from investment is less than that of group i workers as their chance to make it to the final selection is less. Therefore, symmetric equilibrium cannot hold when $\sigma < 1$. An expectation of such a pre-selection discourages group j workers' investment and induces asymmetric investment levels in equilibrium.

[14] Mialon (2014) considers this problem formally in a framework in which the employer decides $\widehat{\lambda}_i$, and if such a decision is optimal compared to the case of not pre-screening the workers on the basis of their group characteristics.

If σ is sufficiently small, $\widehat{\lambda}_i$ is sufficiently close to 1, regardless of λ_i. Proposition 7 for $\rho \leq \underline{\rho}$ and Proposition 8 for $\rho_h < \rho \leq \overline{\rho}$ show, respectively, that

$$W_{MA}(\widehat{\lambda}_i) > W_{HS}(\widehat{\lambda}_i) = W_{HS}(\lambda_i) \text{ and } W_{EA}(\widehat{\lambda}_i) > W_{LS}(\widehat{\lambda}_i) = W_{LS}(\lambda_i),$$

for a $\widehat{\lambda}_i$ sufficiently close to 1. Thus, the employer's payoff improves upon implementation of a biased pre-selection $\sigma < 1$. Proposition 10 summarizes this result.

Proposition 10 *If $\rho \leq \underline{\rho}$ or $\rho_h < \rho \leq \overline{\rho}$, the employer has an incentive to exclude some of job candidates based on their group identity in the pre-selection stage to induce a discriminatory equilibrium.*

Note that this result holds even if λ_i is not close to 0 or 1. To see this, suppose that $\widehat{\lambda}_i > 1/2$ whereas $\lambda_i = 1/2$. This means that, because the employer pre-selects mainly the workers of group i, group j workers do not receive a fair chance to be considered for a position, although the population is equally divided. If this is anticipated, because group j workers' expected probability of being considered for a job is very low, not many of them would invest. This enhances group i workers' incentive to invest. Hence, discriminatory equilibrium can become salient based on the workers' rational beliefs of a biased $\widehat{\lambda}_i$. The equal opportunity protection law in the U.S. mainly guarantees the fairness of opportunity in this pre-selection stage.

Alternatively, we can consider the possibility that the employer commits to a particular selection rule $(\widehat{\varphi}_A, \widehat{\varphi}_B)$ whenever the signals are (M, M) from the two group workers. That is, suppose that the employer ponders whether to commit to selecting group A and group B workers with probabilities of $\widehat{\varphi}_A$ and $\widehat{\varphi}_B$ whenever (M, M) regardless of his beliefs about the two groups' qualifications. For example, the employer may decide to always select a worker from group A when workers from the two groups have the same signals (M, M), i.e., $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$, even if he expects that group B may have more qualified workers.[15] This rule differs from (3)

[15] When $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$, the implication is that the employer hires group A workers for given equally qualifying signals from both group workers. A public commitment to such a rule is illegal, whereas it is always possible for the employer to stick to such a selection rule unopenly.

in our main analysis in Section 2, in which the employer's decision, upon observing workers of two different groups with the same signal M, depends on his beliefs about the two groups' qualifications. Thus, the equilibria in Section 2 involve the employers' beliefs that are consistent with the actual investment levels of workers. Such a "rational expectation" results in statistical discrimination coined by Arrow (1972, 1973). In contrast, the tie-breaking rule in this section implies that the employer commits to not incorporating his beliefs about the two group workers' qualifications when the signals are (M, M).

When this type of commitment is expected, it alters the workers' investment incentives and the equilibrium behavior. In Appendix, we describe in detail how the tie-breaking rule changes the workers' investment behavior. For the purpose of identifying the employer's incentive for discrimination, we consider, in particular, two cases: (1) when the rule favors one group, $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$ or $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (0, 1)$, and (2) when the rule is impartial, $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1/2, 1/2)$. Based on the analyses in these two cases, Proposition 11 shows that, if $\rho \leq \underline{\rho}$, the unfair tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$ is preferred to impartial rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1/2, 1/2)$, whereas if $\rho_h < \rho \leq \overline{\rho}$, no particular rule is preferred.

Proposition 11 *(i) Suppose $\rho \leq \underline{\rho}$. A commitment to $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$ uniquely selects* **MA***, whereas a commitment to $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1/2, 1/2)$ uniquely selects* **HS**. *If $\lambda_A \in (0, 1)$ is sufficiently close to 1, the employer's expected payoff is higher under the unfair rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$ than the fair rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1/2, 1/2)$.*

(ii) Suppose $\rho_h < \rho \leq \overline{\rho}$. Neither of the tie-breaking rules results in a unique equilibrium. The expected payoff under the unfair tie-breaking rule is not necessarily higher than the payoff under the fair tie-breaking rule.

By combining the results from Proposition 11 with those of Propositions 5 and 7, we find that, when $\rho \leq \underline{\rho}$, the employer has an incentive to commit to a tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$ that induces a unique discriminatory equilibrium in the parameter range. Proposition 12 summarizes this result.

Proposition 12 *Suppose $\rho \leq \underline{\rho}$. If $\lambda_i \in (0,1)$ is sufficiently close to 1, the employer's expected payoff from committing to $(\widehat{\varphi}_i, \widehat{\varphi}_j) = (1,0)$ is at least as high as the payoff without the tie-breaking rule. Thus, the employer has an incentive to commits to the tie-breaking rule, which induces a unique asymmetric equilibrium* **MA** *with $\alpha_i > \alpha_j$.*

On the other hand, if $\rho_h < \rho \leq \overline{\rho}$, there is no such incentive, since a commitment to a particular tie-breaking rule does not necessarily induce a unique asymmetric equilibrium.

4 Policy and Welfare Implications

Our discussion in the previous section sheds light on the important roles of equal opportunity laws in shaping workers' expectation of the opportunities to obtain a return from their investment. If λ_i is biased toward one group, the employers may prefer discrimination. Moreover, even if the true population parameter is $\lambda_i \approx 1/2$, if asymmetric equilibrium is preferable to them, the employers have an incentive to choose $\widehat{\lambda}_i$ so that $\widehat{\lambda}_i > \lambda_i$. Also, with an unfair tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B)$, the entire dynamics of the game could change as it would alter the workers' expectations of the return from investment.

This shows why equal opportunity protection is particularly important in the context of a competitive hiring process. Equal opportunity laws help to secure a level playing field for workers from all groups, thus ensuring that the equilibrium is more likely to be non-discriminatory. Equal opportunity laws declare it illegal to make "employment decisions based on stereotypes or assumptions of the abilities, traits, or performance of individuals of a certain sex, race, age, religion, or ethnic group, or individuals with disabilities, [....]"[16] Interpretation of the laws differs from state to state. In some states, affirmative action programs are often used to restore a balance between minority and majority representations in the workplace by encouraging employers to treat minority workers more favorably. This induces $\widehat{\lambda}_i \approx 1/2$. In other states, equal opportunity protection simply means fair judging, when other things are equal, which ensures a fair tie-

[16] http://www.eeoc.gov

breaking rule $\widehat{\varphi}_i = 1/2$ when appropriate.

Our analysis shows that, as a result of the laws, an expectation of favorable conditions or fair opportunities will enhance minority workers' incentive for investment. This makes symmetric equilibrium more likely and preferable to the employers. In this way, the laws guarantee fair opportunities for capable minority workers who, without such laws, would have few prospects, because of their self-fulfilling expectation of unfavorable returns from their investment based on their low level of representation in the population.

In this framework, the problem of wasteful investment is caused by a shortage of available positions, regardless of workers' qualifications. Therefore, discrimination cannot be the solution to the problem of wasteful human capital investment. Instead, creating more positions in the economy would be a more effective solution for both discrimination and wasteful investment. More jobs would encourage workers' investment and engage more qualifying individuals. In a similar framework, Mialon (2014) shows that prejudice is less likely to occur as the shortage of positions declines.

5 Concluding Remarks

We provide a model of discrimination in employment when there is competition between two groups of workers. We show that, when facing competition among workers, employers may seek discriminatory equilibrium.

A comparison of each employer's payoffs under symmetric equilibrium and asymmetric equilibrium shows the role of discrimination in this framework. Given that equal treatment of the two groups is equivalent to having within-group interactions only, discrimination would be preferred only when it helps to divide the population into groups that act differently. In order to divide the population into groups, group-specific traits are necessary, no matter how irrelevant they are, since it is impossible without them to systematically differentiate within the same group. Thus, group-specific traits are used as a means to facilitate such a division.

Dividing the population into groups helps the employer because it can lower overall risk.

The employer faces a risk when hiring a worker with a noisy signal M. The risk when two groups of workers have different levels of expected probability of qualification differs from the risk encountered when the levels are the same. The employer prefers discrimination when the risk is lower as a result of creating different expected probabilities for the two groups.

What makes unequal treatment of potentially identical groups profitable is the curvature of the employer's total expected payoff $W(\alpha_i, \alpha_j, \lambda_i)$ and the different size of each group λ_i. In this framework, $W(\alpha_i, \alpha_j, \lambda_i)$ becomes non-linear because workers are in competition. As the degree of competition grows, $W(\alpha_i, \alpha_j, \lambda_i)$ is expected to become more non-linear, indicating that the employer's payoffs become more sensitive to the spread of risks across groups in competitive signaling. Therefore, the preference for discrimination due to the benefit of a spread of risk is intrinsic in the case of competitive signaling, unlike Moro and Norman (2004).

Although we consider only two groups in a population for the sake of simplicity, the qualitative results of the current model will hold even when the model is extended to the case of many groups. One of the main reasons is that, although more than two types of group indices are available, employers may not want to utilize all of them for the purpose of discrimination. Given that treating all groups equally is an option that is always feasible for employers, it is essentially up to them to determine how many of the available indices to use for differentiation and discrimination. When discrimination is preferred, the optimal number of group indices to use will be chosen so as to maximize the effect of introducing differential investment incentives on the "effectively" differentiated groups in the population. If the current observation of discrimination (as in the case of gender or race discrimination) reflects such an optimization process, it implies that the optimal number of effectively differentiated groups is often two. This enhances the generalizability of the results of our paper.

We show how employers can influence the equilibrium selection by their choice of finalist and tie-breaking rules. Thus, the employers' preference for discrimination is of significant importance in determining the equilibrium. We argue that equal opportunity laws play important roles in securing a level playing field and a fair incentive mechanism for workers.

Appendix: Proofs

5.1 Symmetric equilibria

First, let us show that β is an increasing function of k_i and a strictly decreasing function of k_j.

Lemma 1 *β is an increasing function of k_i given a fixed k_j and a strictly decreasing function of k_j given a fixed k_i.*

Proof. Consider (4). Since $F(\cdot)$ is strictly increasing, it is sufficient to show that β is an increasing function of α_i given a fixed α_j and a strictly decreasing function of α_j given a fixed α_i. For any pair $\alpha_i' > \alpha_i$, $(q-u)\mathbf{1}_{\{\alpha_i' \geq \alpha_s\}} \geq (q-u)\mathbf{1}_{\{\alpha_i \geq \alpha_s\}}$ since $q > u$, and $\varphi(\alpha_i', \alpha_j) \geq \varphi(\alpha_i, \alpha_j)$ given α_j. The first term is a strictly decreasing function of α_j:

$$p_H(\alpha_j)\frac{1}{2} + p_M(\alpha_j) + p_L(\alpha_j) = -\frac{1}{2}\alpha_j(1-q) + 1.$$

For any pair $\alpha_j' > \alpha_j$, we have $\varphi(\alpha_i, \alpha_j') \leq \varphi(\alpha_i, \alpha_j)$ given α_i, and

$$\begin{aligned} & p_M(\alpha_j)\varphi(\alpha_i, \alpha_j) + p_L(\alpha_j) \\ & = -\alpha_j[(1-u) - (q-u)\varphi(\alpha_i, \alpha_j)] + u\varphi(\alpha_i, \alpha_j) + (1-u), \end{aligned}$$

where $(1-u) - (q-u)\varphi(\alpha_i, \alpha_j) > 0$ for all (α_i, α_j), which shows the latter. ■

Proof of Proposition 1. First, $\beta_l(0)v = (1-q)[1 - \frac{1}{2}F(0)(1-q)] > 0$ and $\beta_l(\bar{c})v = (1-q)[1 - \frac{1}{2}F(\bar{c})(1-q)]v < \bar{c}$ since $v < \bar{c}$. Then, from Lemma 1, there exists $k_l \in (0, \bar{c})$ such that $\beta_l(k_l)v = k_l$. Similarly, for β_h, we have $\beta_h(0)v > \beta_l(0)v > 0$ and $\beta_h(\bar{c})v < \bar{c}$, so there exists $k_h \in (0, \bar{c})$ such that $\beta_h(k_h)v = k_h$. Furthermore, k_h and k_l are unique, and $k_h > k_l$ because $\beta_h(k) > \beta_l(k)$, and β_l and β_h are strictly decreasing functions of k.

If $k_s \leq k_l$, the unique fixed point of β_l, k_l, cannot be attained, and since $k_s \leq k_l < k_h$, k_h can be attained. If $k_s > k_h$, the unique fixed point of β_h, k_h, cannot be attained, and since $k_s > k_h > k_l$, k_l can be attained. If $k_l < k_s \leq k_h$, since $k_l < k_s$ and $k_s \leq k_h$, both can be attained. ■

5.2 Asymmetric equilibria

In asymmetric equilibrium, $k_A \neq k_B$. If $k_i \geq k_s$, the net increase in the winning probability for group i workers defined in (4) now depends on whether $k_i > k_j$ or $k_i < k_j$, through $\varphi(\alpha_i, \alpha_j)$ in the second term $S_i = M$, whereas the term with $S_i = M$ disappears if $k_i < k_s$. So, we need to consider two cases, $k_i \geq k_s$ or not. For $k_i \geq k_s$, let $G_{iu} : [0, \bar{c}]^2 \rightarrow [0,1]$ and $G_{id} : [0, \bar{c}]^2 \rightarrow [0,1]$ be the expected payoff of worker i when $k_i > k_j$ and when $k_i < k_j$, respectively. That is,

$$G_{iu}(k_i, k_j) \equiv \lambda_i \beta_h(k_i) + (1-\lambda_i)\beta_u(k_j); \tag{15}$$

$$G_{id}(k_i, k_j) \equiv \lambda_i \beta_h(k_i) + (1-\lambda_i)\beta_d(k_j), \tag{16}$$

where for a given k, $\beta_u : [0,1] \to [0,1]$ and $\beta_d : [0,1] \to [0,1]$ are defined as

$$\begin{aligned}\beta_u(k) &\equiv (1-q)\left[P_H(k)\frac{1}{2} + P_M(k) + P_L(k)\right] + (q-u)[P_M(k) + P_L(k)];\\ \beta_d(k) &\equiv (1-q)\left[P_H(k)\frac{1}{2} + P_M(k) + P_L(k)\right] + (q-u)P_L(k).\end{aligned}$$

The first part of $G_{iu}(k_i,k_j)$ or $G_{id}(k_i,k_j)$, β_h, shows group i worker's winning probability from competition against a same group worker, whereas the second parts, β_u and β_d, represent the probability from competition against a worker from a different group.

When $k_i < k_s$, on the other hand, we define $G_{il}(k_i,k_j)$ as follows.

$$G_{il}(k_i,k_j) \equiv \lambda_i \beta_l(k_i) + (1-\lambda_i)\beta_l(k_j), \tag{17}$$

When a group i worker is paired with a worker from the same group with a probability λ_i, it necessarily induces a symmetric equilibrium. Thus, $\beta_h(k_i)$ and $\beta_l(k_i)$ coincide with those defined in (8) or (9).

Note that for each k, the following relationship holds:

$$\beta_u(k) > \beta_h(k) > \beta_d(k) > \beta_l(k). \tag{18}$$

This relationship in (18) shows the effect of inter-group competition φ on group i workers' incentive to invest. Winning a competition against group j workers enhances the incentive to invest.

To derive the fixed points when $k_A \neq k_B$ and to prove Proposition 2, we will need the results from the following two Lemmas 2 and 3. It is useful to consider an implicit function $g_{i\gamma} : [0,\bar{c}] \to (0,\bar{c})$ such that $k_i = g_{i\gamma}(k_j)$ for any given k_j and each possible case of $\gamma \in \{u,d,l\}$.

Lemma 2 *For each $\gamma \in \{u,d,l\}$, there exists a unique differentiable and strictly decreasing function $g_{i\gamma}$ that satisfies*

$$G_{i\gamma}(g_{i\gamma}(k_j), k_j)\, v = g_{i\gamma}(k_j). \tag{19}$$

Proof of Lemma 2. We prove only for g_{id} since the other proofs can be obtained in a similar way. First, show that for each $k_j \in [0,\bar{c}]$, there exists $k_i \in (0,\bar{c})$ such that $G_{id}(k_i,k_j)\, v = k_i$. From $G_i(0,\bar{c}) = G_{il}(0,\bar{c})$,

$$\begin{aligned}0 &< G_{il}(0,\bar{c})\, v = [\lambda_i \beta_l(0) + (1-\lambda_i)\beta_l(\bar{c})]v\\ &< [\lambda_i \beta_h(0) + (1-\lambda_i)\beta_d(\bar{c})]v = G_{id}(0,\bar{c})\, v.\end{aligned}$$

Since β_d is strictly decreasing, for each $k_j \in [0,\bar{c}]$, $G_{id}(0,k_j)\, v > 0$. From $G_i(\bar{c},0) = G_{iu}(\bar{c},0)$,

$$\begin{aligned}\bar{c} &> G_{iu}(\bar{c},0)\, v = [\lambda_i \beta_h(\bar{c}) + (1-\lambda_i)\beta_u(0)]v\\ &> [\lambda_i \beta_h(\bar{c}) + (1-\lambda_i)\beta_d(0)]v = G_{id}(\bar{c},0)\, v.\end{aligned}$$

Since β_d is strictly decreasing, for each $k_j \in [0, \overline{c}]$, $G_{id}(\overline{c}, k_j)\, v < \overline{c}$. Note

$$\frac{d[G_{id}(k_i, k_j)\, v - k_i]}{dk_i} = \lambda_i \beta_h(k_i) - 1 < 0.$$

By the implicit function theorem,

$$g'_{id}(k_j) = \frac{v(1-\lambda_i)\, \beta'_d(k_i)}{1 - v\lambda_i \beta'_h(k_j)} < 0.$$

■

To show how an asymmetric treatment of two groups affects their investment incentives differently, let us define several reference points that resembles the symmetric cutoffs under different incentives, $\gamma = \{u, d, l\}$. These reference points will be used later to establish the location of asymmetric equilibrium allocations.

Lemma 3 *Let k_{iu}, k_{jd}, and k_{jl} be the fixed points of $G_{id}(k_i, k_i)\, v = k_i$, $G_{jd}(k_j, k_j)\, v = k_j$, and $G_{jl}(k_j, k_j)\, v = k_j$, respectively. Then,*

(i) $k_{iu} > k_{jd} > k_{jl} = k_l$

(ii) For any given $k \in [0, \overline{c}]$, $g_{jd}(k) > g_{jl}(k)$

Proof of Lemma 3. *(i)* Suppose $k_{jd} \geq k_{iu}$. By construction, this implies

$$\begin{aligned} & \lambda_j \beta_h(k_{jd}) + (1-\lambda_j)\, \beta_d(k_{jd}) \geq \lambda_i \beta_h(k_{iu}) + (1-\lambda_i)\, \beta_u(k_{iu}) \\ \Leftrightarrow\ & (1-\lambda_i)\, \beta_h(k_{jd}) + \lambda_i \beta_d(k_{jd}) \geq \lambda_i \beta_h(k_{iu}) + (1-\lambda_i)\, \beta_u(k_{iu}), \end{aligned}$$

and since β_h and β_d are decreasing,

$$\begin{aligned} & (1-\lambda_i)\, \beta_h(k_{iu}) + \lambda_i \beta_d(k_{iu}) \geq \lambda_i \beta_h(k_{iu}) + (1-\lambda_i)\, \beta_u(k_{iu}) \\ \Leftrightarrow\ & (1-\lambda_i)\, [\beta_h(k_{iu}) - \beta_u(k_{iu})] + \lambda_i [\beta_d(k_{iu}) - \beta_h(k_{iu})] \geq 0, \end{aligned}$$

which contradicts $\beta_h(k) < \beta_u(k)$ and $\beta_d(k) < \beta_h(k)$. Similarly, we can prove that $k_{jd} > k_{jl}$.

(ii) Since for each $k \in [0, \overline{c}]$, $\beta_h(k) > \beta_d(k) > \beta_l(k)$,

$$\beta_d(k_i) = \frac{g_{jd}(k_i)}{(1-\lambda_j)\, v} - \frac{\lambda_j}{(1-\lambda_j)} \beta_h(g_{jd}(k_i))$$

implies

$$\beta_l(k_i) < \frac{g_{jd}(k_i)}{(1-\lambda_j)\, v} - \frac{\lambda_j}{(1-\lambda_j)} \beta_l(g_{jd}(k_i)).$$

Since $\frac{k_j}{(1-\lambda_j)v} - \frac{\lambda_j}{(1-\lambda_j)} \beta_h(k_j)$ is a strictly increasing function of k_j, given each k_i, we must have $g_{jl}(k_i) < g_{jd}(k_i)$. ■

Based on the results from Lemmas 2 and 3, we can prove Proposition 2.

Proof of Proposition 2. **Step 1**. Show that there exist (k_i^e, k_j^e) and (k_i^m, k_j^m).

Case 1. **MA** It follows from Lemma 2 and Lemma 3 that $k_{iu} > k_{jd} = g_{jd}(k_{jd}) > g_{jd}(k_{iu})$, which in turn implies

$$g_{iu}^{-1}(k_{iu}) - g_{jd}(k_{iu}) > 0.$$

On the other hand, since g_{iu} is strictly decreasing, $g_{iu}(0)$ is the maximum of g_{iu}, and $g_{iu}(0) < \overline{c}$. By Lemma 2,

$$g_{iu}^{-1}(g_{iu}(0)) - g_{jd}(g_{iu}(0)) < 0.$$

The continuity of g_{iu} and g_{jd} entails that there exists $k_i^m \in (k_{iu}, \overline{c})$ such that

$$g_{iu}^{-1}(k_i^m) - g_{jd}(k_i^m) = 0.$$

Given k_i^m, the value of $g_{iu}^{-1}(k_i^m)$ is k_j^m, which must be in $(0, \overline{c})$. Hence, given (k_i^m, k_j^m),

$$G_{iu}(k_i^m, k_j^m)v = k_i^m \text{ and } G_{jd}(k_j^m, k_i^m)v = k_j^m.$$

Case 2. **EA** Since g_{iu}^{-1} is strictly decreasing, and $k_{iu} = g_{iu}^{-1}(k_{iu})$, given $k_i^m > k_{iu}$, we have $k_i^m > k_j^m$. By Lemma 3,

$$0 = g_{iu}^{-1}(k_i^m) - g_{jd}(k_i^m) < g_{iu}^{-1}(k_i^m) - g_{jl}(k_i^m).$$

On the other hand, $g_{iu}^{-1}(g_{iu}(0)) - g_{jl}(g_{iu}(0)) < 0$. The continuity of g_{iu} and g_{jl} implies that there exists $k_i^e \in (k_i^e, \overline{c})$ such that

$$g_{iu}^{-1}(k_i^e) - g_{jl}(k_i^e) = 0.$$

Hence, given (k_i^e, k_j^e),

$$G_{iu}(k_i^e, k_j^e)v = k_i^e \text{ and } G_{jl}(k_j^e, k_i^e)v = k_j^e.$$

Since g_{iu}^{-1} is strictly decreasing, and $k_{iu} = g_{iu}^{-1}(k_{iu})$, given $k_i^e > k_{iu}$, we have $k_i^i > k_j^i$.

Step 2. Show the uniqueness and the characterization.

Case 1. **MA** In equilibrium, $g_{iu}^{-1}(k_i^m) - g_{jd}(k_i^m) = 0$ or $g_{iu}(g_{jd}(k_i^m)) - k_i^m = 0$. Consider $d[g_{iu}(g_{jd}(k_i)) - k_i]/dk_i$. Given $\lambda_i = 1 - \lambda_j$,

$$\begin{aligned}
& [g_{iu}(g_{jd}(k_i))]' - 1 \\
= \; & \frac{v(1-\lambda_i)\beta_u'(k_j)}{1 - v\lambda_i\beta_h'(k_i)} \frac{v\lambda_i\beta_d'(k_i)}{1 - v(1-\lambda_i)\beta_h'(k_j)} - 1 \\
= \; & \frac{v^2\lambda_i(1-\lambda_i)\left\{\beta_u'(k_j)\beta_d'(k_i) - \beta_h'(k_i)\beta_h'(k_j)\right\} - 1 + v\lambda_i\beta_h'(k_i) + v(1-\lambda_i)\beta_h'(k_j)}{[1 - v\lambda_i\beta_h'(k_i)][1 - v(1-\lambda_i)\beta_h'(k_j)]} < 0,
\end{aligned}$$

where $\beta_h'(\cdot) < 0$ and $\beta_u'(k_j)\beta_d'(k_i) - \beta_h'(k_i)\beta_h'(k_j) = -\frac{1}{4}f(k_i)f(k_j)(q-u)^4 < 0$ for all $k_i > k_j$.

Case 2. **EA** In equilibrium, $g_{iu}^{-1}(k_i^e) - g_{jl}(k_i^e) = 0$ or $g_{iu}(g_{jl}(k_i^e)) - k_i^e = 0$. Consider

$d[g_{iu}(g_{jl}(k_i)) - k_i]/dk_i$. Given $\lambda_i = 1 - \lambda_j$,

$$\begin{aligned}
& [g_{iu}(g_{jl}(k_i))]' - 1 \\
= & \frac{v(1-\lambda_i)\beta_u'(k_j)}{1 - v\lambda_i\beta_h'(k_i)} \frac{v\lambda_i\beta_l'(k_i)}{1 - v(1-\lambda_i)\beta_l'(k_j)} - 1 \\
= & \frac{v^2\lambda_i(1-\lambda_i)\left\{\beta_u'(k_j)\beta_l'(k_i) - \beta_h'(k_i)\beta_l'(k_j)\right\} - 1 + v\lambda_i\beta_h'(k_i) + v(1-\lambda_i)\beta_l'(k_j)}{[1 - v\lambda_i\beta_h'(k_i)][1 - v(1-\lambda_i)\beta_l'(k_j)]} < 0,
\end{aligned}$$

where $\beta_l'(\cdot) < 0$, $\beta_h'(\cdot) < 0$ and $\beta_u'(k_j)\beta_l'(k_i) - \beta_h'(k_i)\beta_l'(k_j) = -\frac{1}{4}f(k_i)f(k_j)(1-q)^2[(1-u)^2 - (1-q)(q-2u+1)] < 0$ for all $k_i > k_j$. Note that both (k_i^m, k_j^m) and (k_i^e, k_j^e) are on the graph $k_i = g_{iu}(k_j)$ where g_{iu} is strictly decreasing. Hence, $k_i^e > k_i^m$ implies $k_j^m > k_j^e$. Thus, we have

$$0 < k_j^e < k_j^m < k_i^m < k_i^e < \overline{c}.$$

■

Proof of Proposition 3. Consider (15), (16) and (17). If $k_s \leq k_j^e$, the fixed point (k_i^e, k_j^e) cannot be attained, and since $k_s \leq k_j^m < k_i^m < k_i^e$, (k_i^m, k_j^m) can be attained. If $k_j^m < k_s \leq k_i^e$, the fixed point (k_i^m, k_j^m) cannot be attained, and since $k_j^e < k_j^m < k_s \leq k_i^e$, (k_i^e, k_j^e) can be attained. If $k_j^e < k_s \leq k_j^m$, since $k_j^e < k_s < k_i^e$ and $k_s \leq k_j^m < k_i^m$, both can be attained. Lastly, if $k_s > k_i^e$, neither can be attained. ■

5.3 Synthesis

Proof of Proposition 4. *(i)* First, we show $k_{iu} > k_h$. Suppose $k_{iu} \leq k_h$. This implies that $\lambda_i\beta_h(k_{iu}) + (1-\lambda_i)\beta_u(k_{iu}) \leq \lambda_i\beta_h(k_h) + (1-\lambda_i)\beta_h(k_h)$, and since β_h is decreasing, we have $\lambda_i\beta_h(k_{iu}) + (1-\lambda_i)\beta_u(k_{iu}) \leq \lambda_i\beta_h(k_{iu}) + (1-\lambda_i)\beta_h(k_{iu})$, so that $\beta_u(k_{iu}) \leq \beta_h(k_{iu})$, which contradicts $\beta_u(k_{iu}) > \beta_h(k_{iu})$. Then, from the proof of Proposition 3, $k_h < k_{iu} < k_i^m$. Similarly, we show $k_{jd} < k_h$. Suppose $k_{jd} \geq k_h$. This implies $(1-\lambda_j)\beta_h(k_{jd}) + \lambda_j\beta_d(k_{jd}) \geq (1-\lambda_j)\beta_h(k_h) + \lambda_j\beta_h(k_h)$, and since β_h is decreasing, we have $(1-\lambda_j)\beta_h(k_{jd}) + \lambda_j\beta_d(k_{jd}) \geq (1-\lambda_j)\beta_h(k_{jd}) + \lambda_j\beta_h(k_{jd})$, so that $\beta_d(k_{jd}) \geq \beta_h(k_{jd})$, which contradicts $\beta_d(k_{jd}) < \beta_h(k_{jd})$. Since g_{jd} is strictly decreasing, and $g_{jd}(k_{jd}) = k_{jd}$, $k_h > k_{jd} > k_j^m$.

(ii) Similarly, the proof of Proposition 3 and the property of g_{jl} can show the result. ■

Proof of Proposition 5. Proposition 4 entails that $\underline{k} < k_l < k_h < \overline{k}$. Then, from (2), $\overline{\rho}$ can be defined as a value at which

$$\overline{\rho} \equiv \frac{uF(\overline{k})}{uF(\overline{k}) + q(1 - F(\overline{k}))}, \tag{20}$$

and similarly, at ρ_h and $\underline{\rho}$,

$$\rho_h \equiv \frac{uF(k_h)}{uF(k_h) + q(1 - F(k_h))}, \text{ and } \underline{\rho} \equiv \frac{uF(\underline{k})}{uF(\underline{k}) + q(1 - F(\underline{k}))}.$$

Proposition 1 and Proposition 3 establish the results. ■

5.4 Discrimination

Proof of Proposition 6. Case 1. Let $\alpha_i = \alpha_j = \alpha \geq \alpha_s$. Then, $\Gamma(\alpha_i, \alpha_j) = \mu(\alpha) R - v$ and $\mu(\alpha_i) = \mu(\alpha_j) = \mu(\alpha)$, so from $U(\alpha_i, \alpha_j)$ in (10), we have

$$V(\alpha) = U(\alpha, \alpha) = \{\Pr(H \vee H) + [P(M, M) + P(M, L) + P(L, M)]\,\mu(\alpha)\} R - [1 - P(L, L)]\, v,$$

where

$$\begin{aligned} \Pr(H \vee H) &= 2p_H(\alpha) - [p_H(\alpha)]^2, \\ [P(M, M) + P(M, L) + P(L, M)] &= [p_M(\alpha)]^2 + 2p_M(\alpha)\, p_L(\alpha), \\ 1 - P(L, L) &= 1 - [p_L(\alpha)]^2. \end{aligned}$$

Hence, the first derivative of $V(\alpha)$ yields

$$V'(\alpha) = (2 - 2\alpha + 2\alpha qu - qu)\, R - 2(1 - \alpha)(1 - u)^2 v. \tag{21}$$

For the two end points $\alpha = 0$ and $\alpha = 1$,

$$V'(0) = (2 - qu)R - 2(1 - u)^2 v > 0 \text{ and } V'(1) = quR > 0,$$

since $R - v > 0$ and $(2 - qu) - 2(1 - u)^2 = -qu + 4u - 2u^2 > 0$. In addition, the second derivative of $V(\alpha)$ yields

$$V''(\alpha) = -2[R(1 - qu) - v(1 - u)^2] < 0 \text{ for all } \alpha \in [0, 1]$$

where $R - v > 0$ and $(1 - qu) - (1 - u)^2 > 0$. This implies that $V'(\alpha) > 0$ for all $\alpha \in [0, 1]$.

Case 2. Let $\alpha_i = \alpha_j = \alpha < \alpha_s$. Then, from $U(\alpha_i, \alpha_j)$, we have $V(\alpha) = U(\alpha, \alpha) = \Pr(H \vee H)(R - v)$. Hence,

$$V'(\alpha) = 2(1 - q)[1 - \alpha(1 - q)](R - v) > 0.$$

■

Proof of Proposition 7. Since $F(k)$ is strictly increasing, $\alpha_i^m = F(k_i^m)$ and $\alpha_j^m = F(k_j^m)$, consider the following simultaneous equations:

$$\begin{aligned} \left[\lambda_i \beta_h(k_i^m) + (1 - \lambda_i)\, \beta_u(k_j^m)\right] v &= k_i^m, \\ \left[(1 - \lambda_i)\beta_h(k_j^m) + \lambda_i \beta_d(k_i^m)\right] v &= k_j^m. \end{aligned}$$

The derivative of them w.r.t. λ_i yields

$$\begin{bmatrix} \lambda_i \beta_h'(k_i^m)\, v - 1 & (1 - \lambda_i)\, \beta_u'(k_j^m) v \\ \lambda_i \beta_d'(k_i^m)\, v & (1 - \lambda_i)\, \beta_h'(k_j^m) v - 1 \end{bmatrix} \begin{bmatrix} \frac{dk_i^m}{d\lambda_i} \\ \frac{dk_j^m}{d\lambda_i} \end{bmatrix} = \begin{bmatrix} \beta_u(k_j^m) v - \beta_h(k_i^m)\, v \\ \beta_h(k_j^m) v - \beta_d(k_i^m)\, v \end{bmatrix}$$

The determinant gives

$$
\begin{aligned}
&\det\begin{bmatrix} \lambda_i\beta_h'\left(k_i^m\right)v-1 & \left(1-\lambda_i\right)\beta_u'(k_j^m)v \\ \lambda_i\beta_d'\left(k_i^m\right)v & \left(1-\lambda_i\right)\beta_h'(k_j^m)v-1 \end{bmatrix} \\
= \;& \lambda_i\left(1-\lambda_i\right)\left[\beta_h'\left(k_i^m\right)\beta_h'(k_j^m)-\beta_u'(k_j^m)\beta_d'\left(k_i^m\right)\right]v^2-\left[\lambda_i\beta_h'\left(k_i^m\right)+\left(1-\lambda_i\right)\beta_h'(k_j^m)\right]v+1>0 \text{ for all } \lambda_i\in[0,1]
\end{aligned}
$$

since $\beta_h'\left(k_i^m\right)\beta_h'(k_j^m)-\beta_u'(k_j^m)\beta_d'\left(k_i^m\right)=\frac{1}{4}f\left(k_i^m\right)f(k_j^m)\left(q-u\right)^4>0$ and $\beta_h'\left(\cdot\right)<0$. Hence, k_i^m and k_j^m are implicit functions of λ_i on $[0,1]$, and they are differentiable. Let $\lambda_i\to 1$. Then,

$$
\lim_{\lambda_i\to 1}\frac{dk_i^m}{d\lambda_i}=\frac{\left[\beta_h\left(k_h\right)-\beta_u(k_h)\right]v}{-\beta_h'\left(k_h\right)v+1}<0,
$$

where as $\lambda_i\to 1$, $(k_i^m,k_j^m)\to(k_h,k_h)$, and $\beta_h\left(k_h\right)-\beta_u(k_h)<0$. ■

Proof of Proposition 8. Since $F\left(k\right)$ is strictly increasing, $\alpha_i^e=F\left(k_i^e\right)$ and $\alpha_j^e=F(k_j^e)$, consider the following simultaneous equations:

$$
\begin{aligned}
\left[\lambda_i\beta_h\left(k_i^e\right)+\left(1-\lambda_i\right)\beta_u(k_j^e)\right]v &= k_i^e, \\
\left[(1-\lambda_i)\beta_l(k_j^e)+\lambda_i\beta_l\left(k_i^e\right)\right]v &= k_j^e.
\end{aligned}
$$

The derivative of them w.r.t. λ_i yields

$$
\begin{bmatrix} \lambda_i\beta_h'\left(k_i^e\right)v-1 & \left(1-\lambda_i\right)\beta_u'(k_j^e)v \\ \lambda_i\beta_l'\left(k_i^e\right)v & \left(1-\lambda_i\right)\beta_l'(k_j^e)v-1 \end{bmatrix}\begin{bmatrix} \frac{dk_i^e}{d\lambda_i} \\ \frac{dk_j^e}{d\lambda_i} \end{bmatrix}=\begin{bmatrix} \beta_u(k_j^e)v-\beta_h\left(k_i^e\right)v \\ \beta_l(k_j^e)v-\beta_l\left(k_i^e\right)v \end{bmatrix}
$$

The determinant gives

$$
\begin{aligned}
&\det\begin{bmatrix} \lambda_i\beta_h'\left(k_i^e\right)v-1 & \left(1-\lambda_i\right)\beta_u'(k_j^e)v \\ \lambda_i\beta_l'\left(k_i^e\right)v & \left(1-\lambda_i\right)\beta_l'(k_j^e)v-1 \end{bmatrix} \\
= \;& \lambda_i\left(1-\lambda_i\right)\left[\beta_h'\left(k_i^e\right)\beta_l'(k_j^e)-\beta_l'\left(k_i^e\right)\beta_u'(k_j^e)\right]v^2-\left[\lambda_i\beta_h'\left(k_i^e\right)+\left(1-\lambda_i\right)\beta_l'(k_j^e)\right]v+1>0 \text{ for all } \lambda_i\in[0,1]
\end{aligned}
$$

since $\beta_h'\left(k_i^e\right)\beta_l'(k_j^e)-\beta_l'\left(k_i^e\right)\beta_u'(k_j^e)=\frac{1}{4}f\left(k_i^e\right)f(k_j^e)\left(1-q\right)^2\left[(1-u)^2-\left(1-q\right)\left(q-2u+1\right)\right]>0$, $\beta_h'\left(\cdot\right)<0$ and $\beta_l'\left(\cdot\right)<0$. Hence, k_i^e and k_j^e are implicit functions of λ_i on $[0,1]$, and they are differentiable. Let $\lambda_i\to 1$. Then,

$$
\lim_{\lambda_i\to 1}\frac{dk_i^e}{d\lambda_i}=\frac{\left[\beta_h\left(k_l\right)-\beta_u(k_l)\right]v}{-\beta_h'\left(k_l\right)v+1}<0,
$$

where as $\lambda_i\to 1$, $(k_i^e,k_j^e)\to(k_l,k_l)$, and $\beta_h\left(k_l\right)-\beta_u(k_l)<0$. ■

Proof of Proposition 9. Suppose that each group workers take turns in making their investment decisions. In particular, let us redefine the equation (4) for group i worker as

$$\begin{aligned}\beta(\alpha_i^{t+1}, \alpha_j^t) = &(1-q)\left[p_H(\alpha_j^t)\frac{1}{2} + p_M(\alpha_j^t) + p_L(\alpha_j^t)\right] \\ &+ \mathbf{1}_{\left\{\alpha_i^{t+1} \geq \alpha_s\right\}}(q-u)[p_M(\alpha_j^t)\varphi(\alpha_i^{t+1}, \alpha_j^t) + p_L(\alpha_j^t)],\end{aligned} \tag{22}$$

for $t = 0, 1, 2, ...,$when the employer makes a hiring decision based on $(\alpha_i^{t+1}, \alpha_j^t)$. Then, the dynamic investment incentive for group i workers for given group j workers' investment α_j^t is

$$\left[\lambda_i \beta\left(\alpha_i^{t+1}, \alpha_i^{t+1}\right) + (1-\lambda_i)\beta\left(\alpha_i^{t+1}, \alpha_j^t\right)\right] v.$$

From Lemma 2, the dynamics of k_i can be written as $k_i^{t+1} = g_{i\gamma^t}(k_j^t)$, where γ^t indicates a class of equilibrium at time t. For given $k_A^{t+2} = g_{A\gamma^{t+2}}(k_B^{t+1})$ and $k_B^{t+1} = g_{B\gamma^{t+1}}(k_A^t)$, the change in k_A from t to $t+2$ is characterized by $k_A^{t+2} = g_{A\gamma^{t+2}}(g_{B\gamma^{t+1}}(k_A^t))$. Then,

$$\begin{aligned}\left|k_A^{t+4} - k_A^{t+2}\right| &= \left|g_{A\gamma^{t+2}}(g_{B\gamma^{t+1}}(k_A^{t+2})) - g_{A\gamma^{t+2}}(g_{B\gamma^{t+1}}(k_A^t))\right| \\ &= [g_{A\gamma^{t+2}}(g_{B\gamma^{t+1}}(x))]' \left|k_A^{t+2} - k_A^t\right|,\end{aligned}$$

where x is between k_A^{t+2} and k_A^t. Hence, for local stability, it is sufficient to show that $[g_{A\gamma^{t+2}}(g_{B\gamma^{t+1}}(x))]' < 1$, where x is a value between k_A^{t+2} and k_A^t. Now, we consider the stability of each equilibrium type.

(i) **LS** : For any local deviation from the equilibrium LS, the two groups' cutoffs are still lower than k_s, and thus, they are in the region where $\gamma^{t+2} = \gamma^{t+1} = l$. It follows from $\beta_l' < 0$ that for all (x_A, x_B), we have

$$\begin{aligned}[g_{Al}(g_{Bl}(x))]' &= \frac{v(1-\lambda)\beta_l'(x_B)}{1 - v\lambda\beta_l'(x_A)} \frac{v\lambda\beta_l'(x_A)}{1 - v(1-\lambda)\beta_l'(x_B)} \\ &= \frac{v(1-\lambda)\beta_l'(x_B)}{1 - v(1-\lambda)\beta_l'(x_B)} \frac{v\lambda\beta_l'(x_A)}{1 - v\lambda\beta_l'(x_A)} < 1.\end{aligned}$$

(ii) **MA** : For any local deviation from the equilibrium **MA**, the two groups' cutoffs around the equilibrium are in the region where $\gamma^{t+2} = u$, $\gamma^{t+1} = d$ if **MA** is characterized by $k_A^m > k_B^m$, or $\gamma^{t+2} = d$, $\gamma^{t+1} = u$ if **MA** is characterized by $k_A^m < k_B^m$. Then, from Proposition 2,

$$[g_{Au}(g_{Bd}(x))]' - 1 < 0,$$

and **MA** is locally stable.

(iii) **EA** : For any local deviation from the equilibrium **MA**, the two groups' cutoffs around the equilibrium are in the region where $\gamma^{t+2} = u$, $\gamma^{t+1} = l$ if **EA** is characterized by $k_A^e > k_B^e$, or $\gamma^{t+2} = l$, $\gamma^{t+1} = u$ if **EA** is characterized by $k_A^e < k_B^e$. Then, from Proposition 2,

$$[g_{Au}(g_{Bl}(x))]' - 1 < 0,$$

and **EA** is locally stable.

(iv) **HS** : Consider a small deviation from **HS** that yields $k_A^t > k_B^{t+1}$ for the two groups' cutoffs. In this case, $\varphi(\alpha_A^t, \alpha_B^{t+1}) = 1$. Hence, the deviation around the equilibrium **HS** moves the cutoffs to the region where $\gamma^{t+2} = u$, $\gamma^{t+1} = d$. It follows from (ii) that the deviation converges to **MA**, which establishes the result. ■

5.5 Impact of tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B)$ on selection

Consider a general case of tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B)$ when (M, M) is observed, where $\widehat{\varphi}_A + \widehat{\varphi}_B = 1$. If $\alpha_i \geq \alpha_s, \alpha_j \geq \alpha_s$, from (4),

$$\widehat{\beta}\,(\alpha_i, \alpha_j, \widehat{\varphi}_i) = (1-q)\,[p_H(\alpha_j)\frac{1}{2} + p_M(\alpha_j) + p_L(\alpha_j)] + (q-u)[p_M(\alpha_j)\widehat{\varphi}_i + p_L(\alpha_j)].$$

The investment incentive of a group i worker increases in $\widehat{\varphi}_i$.

On the other hand, if $\alpha_i \geq \alpha_s, \alpha_j < \alpha_s$, the incentive for group i worker is given as

$$\widehat{\beta}\,(\alpha_i, \alpha_j, \widehat{\varphi}_i) = (1-q)\,[p_H(\alpha_j)\frac{1}{2} + p_M(\alpha_j) + p_L(\alpha_j)] + \mathbf{1}_{\{\alpha_i \geq \alpha_s\}}(q-u)[p_M(\alpha_j) + p_L(\alpha_j)].$$

Note that, in this case, if $\alpha_A \geq \alpha_s, \alpha_B < \alpha_s$, regardless of $(\widehat{\varphi}_A, \widehat{\varphi}_B)$, the employer chooses a group A worker with probability 1 when (M, M) is observed. This holds even if the rule favors group B workers, i.e., $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (0, 1)$. This is because the employer has an incentive to hire a worker of group i with a signal M only if the expected qualification of group i is above the standard, i.e., $\alpha_i \geq \alpha_s$. If group B's qualification is below the standard, the employer incurs losses from hiring the worker. Therefore, the tie-breaking rule is effective only if $\alpha_A \geq \alpha_s, \alpha_B \geq \alpha_s$. Thus, when the rule favors group i workers, the workers gain the advantage of the rule only if $\alpha_i \geq \alpha_s$ for all $i \in \{A, B\}$.

The incentive for group i worker's investment is given as

$$[\lambda_i \widehat{\beta}\,(\alpha_i, \alpha_i, 1/2) + (1-\lambda_i)\widehat{\beta}\,(\alpha_i, \alpha_j, \widehat{\varphi}_i)]v.$$

In equilibrium,

$$G_i(k_i^*, k_j^*)v = k_i^*, \tag{23}$$

where $G_i\,(k_i, k_j) \equiv \lambda_i \widehat{\beta}(F(k_i), F(k_i), 1/2) + (1-\lambda_i)\widehat{\beta}(F(k_i), F(k_j), \widehat{\varphi}_i)$ and $F(k_i) = \alpha_i$.

In the following, we first derive the equilibria when the employer commits to $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$ and $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1/2, 1/2)$, respectively, for each level of ρ (Lemmas 4 and 5). Proposition 11 shows the optimal tie-breaking rule for a given ρ. Then, we examine when the employer would have an incentive to make a commitment to such a tie-breaking rule (Proposition 12). Commitment to $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (0, 1)$ is symmetric to the case of $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$. Even if the rule favors group A workers, it may be possible in equilibrium that group A workers invest less than group B workers, even after the advantage of the tie-breaking rule. In order to determine if such an equilibrium arises, let us call the equilibrium reversed **EA** or reversed **MA**. **EA** and **MA** will refer to the intuitive case in which the favored group invests more.

Lemma 4 first summarizes the equilibria under the tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$.

Lemma 4 *Under the tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$,*

(i) if $\rho > \overline{\rho}$, the unique equilibrium is **LS**,

(ii) if $\rho_h < \rho \leq \overline{\rho}$, equilibrium occurs at **EA**, *reversed* **EA**, *or* **LS**,

(iii) if $\underline{\rho} < \rho \leq \rho_h$, any one of the three types of equilibria among **EA**, **MA**, *reversed* **EA**, *or* **LS** *is possible, and*

(iv) if $\rho \leq \underline{\rho}$, equilibrium occurs at **MA**.

Proof of Lemma 4. In the case of commitment to $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$, there are four possible cases to consider.

Case (i) $k_A, k_B \geq k_s$

In this case, it is either $k_A > k_B$ or $k_A \leq k_B$. When $k_A > k_B$, for **MA**, the equilibrium condition satisfies $g_{Au}^{-1}(k_A) - g_{Bd}(k_A) = 0$ or $g_{Au}(g_{Bd}(k_A)) - k_A = 0$. On the other hand, as the employer commits to a tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$, even if $k_A \leq k_B$, the workers expect the same condition to hold in equilibrium. The fixed points satisfying the equality for this condition constitute the equilibrium. From the proof of Proposition 2, we show that $g_{Au}(g_{Bd}(k_A)) - k_A$ is strictly decreasing. Since the function is decreasing not only for $k_A > k_B$ but also for $k_A \leq k_B$, the fixed point is unique and is **MA**. In particular, since the equality does not hold when $k_A = k_B$, and $g_{Au}(g_{Bd}(k_A)) - k_A$ is strictly decreasing around the point $k_A = k_B$, this proves that there is no symmetric equilibrium. Hence, the tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$ selects only **MA**.

Case (ii) $k_A \geq k_s$, $k_B < k_s$. For **EA**, the equilibrium condition satisfies $g_{Au}(g_{Bl}(k_A)) - k_A = 0$. Since Proposition 2 shows that $g_{Au}(g_{Bl}(k_A)) - k_A$ is strictly decreasing for any other $k_A > k_B$, **EA** is the unique equilibrium under the tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$.

Case (iii) $k_A < k_s$, $k_B \geq k_s$. In this case, group A workers' investment level is below k_s. Thus, even with the tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$, when two different workers emit (M, M), a group B worker wins with probability 1. Hence, the fixed point occurs at the intersection of $[\lambda\beta_l(k_A) + (1 - \lambda)\beta_l(k_B)]\, v = k_A$ and $[(1 - \lambda)\beta_h(k_B) + \lambda\beta_u(k_A)]\, v = k_B$. This results in reversed **EA** where group B workers' investment level is higher than that of group A workers.

Case (iv) $k_A, k_B < k_s$. In this case, the unique equilibrium is **LS**.

The following summarizes these results across the regions of ρ: If $\rho > \overline{\rho}$, the unique equilibrium is **LS**. If $\rho_h < \rho \leq \overline{\rho}$, equilibrium occurs at **EA**, reversed **EA**, or **LS**. If $\underline{\rho} < \rho \leq \rho_h$, any one of the three types of equilibria among **EA**, **MA**, reversed **EA**, or **LS** is possible. Lastly, if $\rho \leq \underline{\rho}$, equilibrium occurs at **MA**. ■

Lemma 4 shows that, compared to the case without a commitment in Proposition 5, the commitment to discriminatory tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$ removes symmetric equilibrium **HS** in the regions (iii) and (iv) where $\rho \leq \rho_h$. One more thing to notice is that, even though the rule favors group A workers, in equilibrium, it may be that $\alpha_A < \alpha_B$. This arises when the players believe $k_A < k_s < k_B$ in the regions (ii) and (iii). In this case, because too few group A workers invest, their signal M is not worthwhile for the employer to consider and, thus, their advantage over group B workers in the tie-breaking rule does not materialize. The results of $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (0, 1)$ are symmetric to this case.

On the other hand, when the tie-breaking rule is $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1/2, 1/2)$, the equilibria are characterized as follows:

Lemma 5 *Under the tie-breaking rule* $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1/2, 1/2)$,

(i) if $\rho > \overline{\rho}$, *the unique equilibrium is* **LS**,

(ii) if $\rho_h < \rho \leq \overline{\rho}$, *equilibrium occurs at* **EA**, *reversed* **EA**, *or* **LS**,

(iii) if $\underline{\rho} < \rho \leq \rho_h$, *any one of the three types of equilibria among* **EA**, **HS**, *or* **LS** *is possible, and*

(iv) if $\rho \leq \underline{\rho}$, *equilibrium is* **HS**.

Proof of Lemma 5. In the case of commitment to $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1/2, 1/2)$, there are four possible cases to consider.

Case (i) $k_A, k_B \geq k_s$. The equilibrium is derived from the following conditions: for $(k_A, k_B) \in [k_s, \overline{c}]^2$,

$$[\lambda\beta_h(k_A) + (1-\lambda)\beta_h(k_B)]\,v = k_A,$$
$$[(1-\lambda)\beta_h(k_B) + \lambda\beta_h(k_A)]\,v = k_B.$$

In the proof of Proposition 1, we have shown that **HS** satisfies the conditions. Moreover, we find that

$$\begin{aligned}
[g_{Ah}(g_{Bh}(x))]' &= \frac{v(1-\lambda)\beta_h'(x_B)}{1-v\lambda\beta_h'(x_A)}\frac{v\lambda\beta_h'(x_A)}{1-v(1-\lambda)\beta_h'(x_B)} \\
&= \frac{v(1-\lambda)\beta_h'(x_B)}{1-v(1-\lambda)\beta_h'(x_B)}\frac{v\lambda\beta_h'(x_A)}{1-v\lambda\beta_h'(x_A)} < 1.
\end{aligned}$$

This indicates that $g_{Ah}(g_{Bh}(k_A)) - k_A$ is strictly decreasing not only at $k_A = k_B$ but also at $k_A \neq k_B$. Hence, the unique fixed point in this range is **HS**.

Case (ii) $k_A \geq k_s$, $k_B < k_s$ and Case (iii) $k_A < k_s$, $k_B \geq k_s$. Since the two cases are symmetric, we analyze (ii) only. In case (ii), even with the tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1/2, 1/2)$, whenever (M, M) is observed, group A workers win with probability 1, because group B workers' investment level is below k_s. The equilibrium occurs at the intersection of $[\lambda\beta_h(k_A) + (1-\lambda)\beta_u(k_B)]\,v = k_A$ and $[(1-\lambda)\beta_l(k_B) + \lambda\beta_l(k_A)]\,v = k_B$. Hence, the tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1/2, 1/2)$ selects only **EA**.

Case (iv) $k_A, k_B < k_s$. As before, in this case, the unique equilibrium is **LS**.

Summarizing these results across the regions of ρ, we have the following: If $\rho > \overline{\rho}$, the unique equilibrium is **LS**. If $\rho_h < \rho \leq \overline{\rho}$, equilibrium occurs at **EA**, reversed **EA**, or **LS**. If $\underline{\rho} < \rho \leq \rho_h$, any one of the three types of equilibria among **EA**, reversed **EA**, **HS**, or **LS** is possible. Lastly, if $\rho \leq \underline{\rho}$, equilibrium occurs at **HS**. ■

Proof of Proposition 11. Combining the results from Lemmas 4 and 5, we find that, if $\rho \leq \underline{\rho}$, the tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$ uniquely selects **MA,** whereas the rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) =$

$(1/2, 1/2)$ uniquely selects **HS.** Proposition 7 shows that $W_{MA}(\lambda_i) > W_{HS}(\lambda_i)$ for a sufficiently high λ. Thus, if $\rho \leq \underline{\rho}$, the unfair tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$ is preferable to the fair rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1/2, 1/2)$. On the other hand, if $\rho_h < \rho \leq \overline{\rho}$, Lemmas 4 and 5 show that both the unfair and fair rules predict the same outcome of multiple equilibria with **EA**, reversed **EA**, and **LS**. Therefore, the employer finds no advantage by committing to either of the two tie-breaking rules. ■

Proof of Proposition 12. Lemma 4 shows that, compared to the case without commitment shown in Proposition 5, commitment to a discriminatory tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$ removes a symmetric equilibrium **HS** in the regions (iii) and (iv) where $\rho \leq \rho_h$. In particular, when $\rho \leq \underline{\rho}$, if not for the tie-breaking rule, a symmetric equilibrium **HS** is also feasible, as well as **MA**. Proposition 7 shows that, for a sufficiently high λ, $W_{MA}(\lambda_i) > W_{HS}(\lambda_i)$. Therefore, in this case, the employer's expected payoff under the unfair rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$ is at least as high as the payoff without the commitment. Thus, when $\rho \leq \underline{\rho}$, if λ is sufficiently high, the employer has an incentive to commit to a tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$ so as to uniquely select an asymmetric equilibrium.

On the other hand, if $\rho_h < \rho \leq \overline{\rho}$, tie-breaking rule $(\widehat{\varphi}_A, \widehat{\varphi}_B) = (1, 0)$ is unable to remove the symmetric equilibrium **LS**. Hence, the employer is unable to influence the selection of asymmetric equilibrium **EA** by committing to a tie-breaking rule. Therefore, the employer will not have an incentive to commit to the tie-breaking rule if $\rho_h < \rho \leq \overline{\rho}$. ■

References

Aigner, D. J. and G.C. Cain (1977), Statistical Theories of Discrimination in Labor Markets, *Industrial and Labor Relations Review*, **30**, 1975-87.

Arrow, K.J. (1972), Models of job discrimination, in A. Pascal, eds., *Racial Discrimination in Economic Life*, Lexington, MA: Lexington Books, 83-102.

Arrow, K.J. (1973), The theory of discrimination, in O. Ashenfelter and A. Rees, eds., *Discrimination in Labor Markets*, Princeton, NJ: Princeton University Press, 3-33.

Becker, G.S. (1971), *The Economics of Discrimination*, Chicago: The University of Chicago Press.

Blume, L. (2005), Learning and statistical discrimination, *American Economic Review* **95**, 118-121.

Burstein, P. (1994), *Equal Employment Opportunity: Labor Market Discrimination and Public Policy*, Aldine Transaction.

Cain, G. (1986), The economic analysis of labor market discrimination: a survey, in O. Ashenfelter and R. Layard, eds., *Handbook of Labor Economics, Vol. I*, New York: Elsevier Science, 693-785.

Chaudhauri, S. and Sethi, R. (2008), Statistical discrimination with peer effects: can integration eliminate negative stereotypes?, *Review of Economic Studies* **75**, 579-596.

Coate, S. and Loury, G.C. (1993), Will affirmative-action policies eliminate negative stereotypes?, *American Economic Review* **83**, 1220-1240.

Cornell, B. and I. Welch (1996), Culture, Information, and Screening Discrimination, *Journal of Political Economy*, **104** (3), 542-571.

England, E. (1992), *Comparable Worth: Theories and Evidence*, Hawthorne, N.Y.: Aldine de Gruyter.

Fang, H. and Moro, A. (2011), *Theories of Statistical Discrimination and Affirmative Action: A Survey,* Handbook of Social Economics, vol (1A), edited by Jess Benhabib, Mattew O. Jackson and Alberto Bisin, North-Holland, 133-200.

Fryer, R.G. (2007), Belief flipping in a dynamic model of statistical discrimination, *Journal of Public Economics* **91**, 1151-1166.

Kreps, D. (1990), Corporate culture and economic theory. In: J. Alt and K. Shepsle (eds.): *Perspectives on Positive Political Economy*, Cambridge, U.K.: Cambridge University Press.

Lang, K. (1986), A Language Theory of Discrimination, *Quarterly Journal of Economics,* **101**, 363-382.

Levin, D. and Smith, J.L. (1994), Equilibrium in Auctions with Entry, *American Economic Review* **84**, 585-599.

Lu, J. (2009), Auction Design with Opportunity Cost, *Economic Theory,* **38 (1)**, 73–103.

Mailath, G.J., Samuelson, L. and Shaked, A. (2000), Endogenous inequality in integrated labor markets with two-sided search, *American Economic Review* **90**, 46-72.

Mialon, S. (2014), Prejudice and Competitive Signaling, Emory University Working Paper series 13-13.

Moro, A. and Norman, P. (2004), A general equilibrium model of statistical discrimination, *Journal of Economic Theory* **114**, 1-30.

Pager, D. and Shepherd, H. (2008), The sociology of discrimination: racial discrimination in employment, housing, credit, and consumer market, *Annual Review of Sociology* **34**, 181-209.

Petersen, T. and Saporta, I. (2004), The opportunity structure for discrimination, *American Journal of Sociology* **109**, 852-901.

Phelps, E.S. (1972), The statistical theory of racism and sexism, *American Economic Review* **62**, 659-661.

Stiglitz, J.E. (1973), Approaches to the economics of discrimination, *American Economic Review (Papers and Proceedings)* **63**, 287-295.