Private v. Public Antitrust Enforcement: A Strategic Analysis

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Abstract. We compare private and public enforcement of the antitrust laws in a simple strategic model of antitrust violation and lawsuit. The model highlights the tradeoff that private firms are initially more likely than the government to be informed about antitrust violations, but are also more likely to use the antitrust laws strategically, to the disadvantage of consumers. Assuming coupled private damages, if the court is sufficiently accurate, adding private enforcement to public enforcement always increases social welfare, while if the court is less accurate, it increases welfare only if the government is sufficiently inefficient in litigation. Pure private enforcement is never strictly optimal. Public enforcement can achieve the social optimum with a fee for public lawsuit that induces efficient information revelation. Private enforcement can also achieve the social optimum with private damages that are efficiently multiplied and decoupled.

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In the U.S., Section 7 of the *Sherman Act of 1890*[1] and Section 4 of the *Clayton Act of 1914*[2] entitle any firm to bring a lawsuit against a competitor for three times the damages suffered from any violation of the antitrust laws. Private enforcement of the antitrust laws is thus explicitly permitted and indeed encouraged, and supplements public enforcement by the Antitrust Division of the U.S. Department of Justice and the Federal Trade Commission. In contrast, in Canada, *The Combines Investigation Act of 1889*[3] contained no provision for private antitrust enforcement until an amendment was added to it in 1976. The amendment only entitles a firm to bring a lawsuit for single (not treble) damages. Moreover, the courts have been ambivalent as to its constitutional validity. As a result, private antitrust enforcement actions have been rare. Similarly, in Europe, the antitrust system has largely discouraged private enforcement, in favor of public enforcement. In both Canada and Europe, policy-makers are now debating whether to change competition laws to encourage more private enforcement.[4] The European Commission (2005) recently released a Green Paper on proposals for new measures to encourage private antitrust enforcement.

Private enforcers have greater incentive to take enforcement action than public enforcers, which may benefit society through additional deterrence. But this is a double-edged sword. Private enforcers also have greater incentive to use the antitrust laws strategically, that is, to use the laws to win in the courts what they were unable to win in honest competition with their rivals. For example, firms may use the antitrust laws to prevent large potential competitors from entering their market, as in the classic case of Utah Pie Co. v. Continental Baking (386 U.S. 685, 1967, U.S. Court of Appeals, 1978).[5] Firms can also use the antitrust laws to prevent their rivals from competing vigorously, extort funds from successful rivals, improve contractual conditions, enforce tacit collusive agreements, respond to existing suits, and prevent hostile takeovers. These strategic uses of the antitrust laws (which often have little to do with promoting efficiency) are explained and documented with many recent U.S. antitrust cases in McAfee and Vakkur (2004) and McAfee, Mialon, and Mialon (2006).

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[1] *An Act to protect trade and commerce against unlawful restraints and monopolies,* ch. 647, 26 Stat. 209 (1890), current version at 15 U.S.C. § 1 et seq. (2003).

[2] *An Act to supplement existing laws against unlawful restraints and monopolies, and for other purposes*, ch. 323, 38 Stat. 730 (1914), current version at 15 U.S.C. § 12 (2003).

[3] *An Act for the prevention and suppression of combinations formed in restraint of trade*, S.C. 1889, c. 41.

[4] Roach and Trebilcock (1996) compare the antitrust enforcement systems in the U.S., Canada, and Australia. Wills (2003) and Ginsburg (2005) compare the antitrust enforcement systems in the U.S. and Europe. These papers also discuss in detail the pros and cons of private antitrust enforcement in general.

[5] Utah Pie is a small firm that produces fresh pies in Salt Lake City, Utah. To counter the arrival of three much larger national competitors that were attempting to penetrate the market by selling frozen pies, Utah Pie initiated an antitrust suit, arguing that the arrival of the new entrants was an attempt to monopolize the market. The Supreme Court ultimately ruled in favor of Utah Pie. The smaller Utah Pie successfully used the antitrust laws to prevent three formidable potential competitors from entering its market. Competition alone would not likely have produced this outcome.

However, private enforcers also have other advantages over public enforcers. Their costs of detecting possible violations and gathering initial evidence are lower. Public enforcers regulate a vast array of industries, and therefore cannot detect anti-competitive practices as easily as private enforcers who experience these practices on a regular basis. In general, private enforcers are better informed about their particular industry. As Shavell (1984) argues, "private parties should generally enjoy an inherent advantage in knowledge" over public regulators. They are naturally in a superior position to estimate the benefits and costs of their own activities and those of their close competitors. "For a regulator to obtain comparable information would often require virtually continuous observation of parties' behavior, and thus would be a practical impossibility" (p. 360). Also, as Brodley (1996) argues, "competitors and takeover targets are ideal litigants in terms of litigation capability because they are likely to have the skill, knowledge of the industry, and motivation to mount a powerful case with speed and precision" (p. 35).

In this paper, we compare private and public antitrust enforcement in a simple strategic model of antitrust violation and lawsuit. The model highlights the tradeoff that firms are initially more likely than the government to be informed about antitrust violations, but are also more likely to use the antitrust laws strategically, to the detriment of consumers. Firms choose whether to take an action that either violates the antitrust laws or improves their own efficiency, the government chooses whether to sue them publicly, and rival firms choose whether to sue them privately. Initially, the rival firms are better informed than the government about whether an illegal or efficient action is taken. The government only wants to sue if an illegal action is taken, but rival firms may want to sue even if an efficient action is taken. The model is solved for its equilibrium outcomes under different enforcement mechanisms (including pure private, pure public, and public combined with private enforcement), which are then compared in terms of social welfare.

Adding private enforcement to public enforcement is always socially beneficial if the court is sufficiently accurate, i.e., likely to rule in favor of the defendant when the defendant is innocent and against the defendant when the defendant is guilty. In this case, firms never strategically abuse the laws, only suing when their competitors have committed an antitrust infraction, so that private enforcement only serves to counter antitrust harm. But if the court is less accurate, adding private enforcement is beneficial only if the government's litigation costs, which depend on its efficiency, are sufficiently high. In this case, firms always sue when their rivals take efficient actions, preferring to take a chance with the courts than suffer a certain loss in market share. Society benefits from private suits only if the government is sufficiently inefficient in litigation and the legitimate private suits outweigh the strategic suits.

We find that the combination of private and public enforcement tends to lead to a greater probability of private than public action, as is observed empirically. In most cases, firms have sufficient incentive to sue if they learn that their rivals have actually violated the antitrust laws. Knowing this, the government has little reason to sue, since it can expect that most of the rightful suits are already being initiated privately. Thus, public enforcement tends to give way to private enforcement when the two are in play. This is consistent with the observation that private antitrust suits have outnumbered public suits in the U.S. by a 9-to-1 ratio from 1970 to 1995.[6]

In the model, the government only has reason to sue if the litigation costs of private firms are very high. But in this case, firms never sue, even if they know that their rivals would otherwise get away with an antitrust violation, so that pure private enforcement yields lower welfare than public enforcement, whether or not the latter is combined with private enforcement, as long as society prefers some public enforcement to no enforcement at all. Moreover, in the majority of cases, where the government has no reason to sue, private combined with public enforcement is equivalent to pure private enforcement. Hence, pure private enforcement is never strictly optimal.

A mechanism of public enforcement can achieve the social optimum with a fee for public lawsuit that induces efficient information revelation. Firms cannot in general credibly reveal their full information to the government at the outset because part of what they know is not verifiable, at least before a lawsuit is initiated and discovery requirements come into effect. However, one way that firms might indirectly reveal their full information is through their willingness to pay a fee to initiate a public lawsuit. The amount of the fee can be set so that firms are willing to pay it to initiate a public lawsuit against a rival firm only if the rival firm's action was illegal. In this case, the fee induces full information revelation. The government is then as well informed as private firms so that mistakes are minimized, and no strategic abuse occurs since only the government, which has the correct incentives (especially if the fee revenue goes to society), sues. Hence, the social optimum is achieved.

The social optimum can also be achieved through a mechanism with private enforcement by efficiently multiplying and decoupling private damages. If the plaintiff wins at trial, the defendant can be required to pay the plaintiff a multiple of the amount of damages that it inflicted on the plaintiff. Moreover, the plaintiff can be made to receive only a fraction of what the defendant pays for an antitrust violation, with the rest going to society. Multiplying damages deters firms from taking illegal actions, while decoupling damages reduces the incentives of firms to strategically abuse the antitrust laws. Therefore, under private enforcement with damages both efficiently

[6] NAFTA Working Group, *Private Actions for Violations of Antitrust Laws*, Appendix A.

multiplied and decoupled, firms take an action if and only if it is legal and sue if and only if their rival took an illegal action, which is the socially optimal outcome.

We also analyze the implementability of optimal private enforcement with multiplied and decoupled damages and optimal public enforcement with a fee for public lawsuit. Less information is required to implement optimal public than optimal private enforcement. On the other hand, optimal private enforcement is implementable in the presence of public enforcement, whereas optimal public enforcement is only implementable in the absence of private enforcement.

The paper proceeds as follows. Section II discusses related literature. Section III develops the model of antitrust violation and suit. Section IV solves the model under pure public enforcement. Section V solves it under private and public, and under pure private, enforcement. Section VI compares mechanisms in terms of social welfare. Section VII shows that public enforcement can achieve the social optimum with a fee for public lawsuit. Section VIII shows that private enforcement can achieve the optimum with damages that are multiplied and decoupled. Section IX analyzes the implementability of the two optimal mechanisms. Section X summarizes and discusses extensions.

II. Related Literature

In a paper written since this one, Segal and Whinston (2006) provide a detailed survey and assessment of the literature on public versus private antitrust enforcement. The literature starts with Becker and Stigler (1974) who argue that free competition among private law enforcers for the damages that are levied against convicted violators could achieve deterrence as efficiently as optimal public enforcement. Landes and Posner (1975) and Posner (1992) challenge the Becker and Stigler argument. Under public enforcement, if the probability of enforcement is equal to one, the penalties should be set equal to the social costs of the illegal activity. By increasing the penalties and reducing the probability of enforcement, society can achieve the same deterrence level at less cost. But under private enforcement, increasing the penalties increases the enforcement probability, as it increases the incentives of private enforcers. Thus, private enforcement can lead to over-deterrence. In contrast, Polinsky (1980) argues that it can lead to under-deterrence of poor offenders because of limited liability, and Garoupa (1997) shows that it can lead to both under-detection and lower accuracy.

These arguments about over and under-deterrence generally assume that private enforcers act legitimately, that is, that they never seek to enforce the law against individuals who have not engaged in the illegal activity. In reality, potential private enforcers may have incentive to behave strategically. This danger is particularly high in the antitrust field because the plaintiffs are often competitors or takeover targets of defendants. They may have an incentive to employ private enforcement strategically, that is, to sue even if they know that their competitors did not violate the antitrust laws.

The prevalence of strategic abuse of antitrust laws by private firms is documented by, among others, Baumol and Ordover (1985), Breit and Elzinga (1985), Shughart II (1990), Brodley (1995), Shavell (1997), McAfee and Vakkur (2004), and McAfee, Mialon, and Mialon (2006). For example, hostile takeover targets often initiate antitrust suits against their acquirers, because such suits create substantial delays that allow the target firms to implement anti-takeover strategies, such as poison pills. If the intended takeover is good for the market, these antitrust actions have a negative effect.

The extent to which firms strategically abuse the antitrust laws under private enforcement also depends crucially on the structure of damage awards in private antitrust cases. The welfare effects of multiplying, and in particular trebling, damages are discussed by Block, Nold, and Sidak (1981), Breit and Elzinga (1985), Easterbrook (1985), Newmark (1988), Kaplow (1992), and Briggs, Huryn, and McBride (1996). Treble damages reduce firms' incentives to violate antitrust laws, but also increase their incentives to use antitrust laws strategically against their rivals.

For example, firms can use the powerful threat of treble damages to extort funds from successful rivals. The actions that are taken to extort money are often resolved through the payment of a *tax on success* for the firms whose positions are sought after by competitors. But taxes on success discourage investment and innovation, which harms consumers (Gentry and Hubbard, 2000). In our model, the welfare-maximizing outcome cannot in general be achieved under private enforcement only by multiplying damages, precisely because multiplying damages encourages firms to sue their successful rivals, which discourages their rivals from taking efficiency-improving actions.

Treble damages can also have perverse effects on consumer enforcement. Block *et al.* (1981) develop a model of the interaction between collusive sellers and competitive buyers under treble damages for price-fixing, which Salant (1987) modifies to take into account that treble damages may have the perverse effect of stimulating demand at any price set by the cartel. The idea is that buyers have incentive to "get damaged" if they expect to get treble damages later. But this effect is weaker if buyers are imperfectly informed about the cartel's costs (Besanko and Spulber, 1990). These models concern enforcement by consumers, while our model focuses on enforcement by competing firms.

Private damages can also be decoupled, in that the amount awarded to the plaintiff differs from that paid by the defendant. This idea is introduced in Schwartz (1980). Polinsky and Che (1991) analyze the effects of decoupling in a tort model. In their model, if the defendant's payment is at the upper bound, decoupling reduces the plaintiff's incentive to sue without affecting the defendant's incentive to exercise care. In other words, decoupling reduces litigation costs without affecting deterrence. Their model does not consider strategic interaction or the possibility of strategic abuse of the laws. We consider the effects of decoupling on the strategic interaction between the plaintiff and defendant in the antitrust context, where the incentive for strategic abuse is especially high. In our model, decoupling reduces deterrence of efficient actions without affecting that of illegal actions. Decoupling reduces the plaintiff's incentive to strategically abuse the laws, which in turn increases the potential defendant's incentive to take efficient actions. Sufficiently multiplying damages ensures that illegal actions are always deterred, while sufficiently decoupling damages ensures that efficient actions are never deterred, so that private enforcement can achieve the socially optimal outcome. Moreover, we show that public enforcement can also achieve the optimum with a fee for public lawsuit that induces information revelation, and we compare public enforcement with a fee to private enforcement with multiplied and decoupled damages in terms of implementability.

III. Model

Consider an industry comprised of two competing firms, firm 1 and firm 2, and a government agency, GOV, in charge of enforcing the antitrust laws on behalf of the public. At time 1, Nature offers firm 1 either a potential action that is pro-competitive, denoted by PC_1, or a potential action that is anti-competitive, denoted by AC_2. The action is type PC_1 with probability p.

At time 2, firm 1 chooses whether to take the action, A or $\neg A$. An action of type AC_2 violates the antitrust laws, harming both competition and consumers, while an action of type PC_1 does not, because it is beneficial to consumers. If firm 1 does not take an action, the game ends. If firm 1 takes an action, at time 3, Nature privately informs firm 2 about the type of firm 1's action, and sends GOV a signal about the type of firm 1's action, which either indicates that the action is type PC_1, a realization denoted by SPC_1, or indicates that the action is type AC_2, denoted by SAC_2. GOV's signal is correct with probability q. It is imperfect, but more often right than wrong, that is, $q > 1/2$. At time 4, firm 2 costlessly complains to GOV about firm 1's action, and if GOV's signal is SAC_2, then GOV chooses whether to sue firm 1 for violating the antitrust laws, S or $\neg S$, knowing the signal but not the type of firm 1's action.

In the basic setup, firm 2 cannot credibly reveal its full information to GOV before GOV chooses whether to sue. The information firms possess about the actions of their rivals is often unverifiable, at least before a lawsuit is filed and discovery rules are in effect. There is no cost to firms of simply complaining to GOV that a rival took an illegal action. Given the strategic incentives of firms, any initial talk that they have with GOV is cheap talk.[7] However, one way that firms might indirectly reveal their full information to GOV is through their willingness to pay a fee for public lawsuit. Section VII extends the model to consider the possibility of information revelation.

GOV never sues if the signal is SPC_1. If GOV does not sue, then at time 5, firm 2 chooses whether to sue firm 1 privately, knowing the type of firm 1's action.[8] If firm 2 does not sue, the game ends. If GOV or firm 2 sues, at time 6, the case proceeds to court, where Nature chooses either the guilty verdict, denoted by G, or the innocent verdict, denoted by I. The court's verdict is correct with probability $r > 1/2$. For simplicity, we initially assume that the verdict is independent of the identity of the plaintiff. Once the verdict is chosen, the game ends.[9]

If firm 1 does not take the action, firm 1, firm 2, and GOV receive payoffs normalized to zero. If firm 1 takes an action, and GOV and firm 2 do not sue, or one of them sues but the verdict is I, then an amount T of profits is transferred from firm 2 to firm 1, and social efficiency increases by x if the action is type PC_1, and decreases by x if the action is type AC_2. The value of x is determined by the harm to competition created by an anticompetitive action, or the loss from preventing a procompetitive action. While, in principle, the effect of a cost increase for firm 2 or a decrease for firm 1 could be different, we take them to be the same for simplicity. If GOV or firm 2 sues and the verdict is G, then the court orders firm 1 to pay compensatory damages to firm 2 in the exact amount T, and to undo its action, which removes the efficiency gain if the action is type PC_1 or the efficiency loss if the action is type AC_2.

Firm 1, firm 2, and GOV's litigation costs are given by L_1, L_2, and L_{GOV}, respectively. For simplicity, we assume $L_1 > L_2$. Antitrust suits often cost more to defend against than to bring against a rival, which contributes to their effectiveness as a strategic tool in attacking rivals.[10] Firm 1's payoff depends positively on the transfer, T, and negatively on its litigation cost, L_1. Firm 2's payoff

[7] Also, firms often hesitate to share their private information with the government before a lawsuit is filed, even if the information is partially verifiable, for fear that it might leak to rival firms.

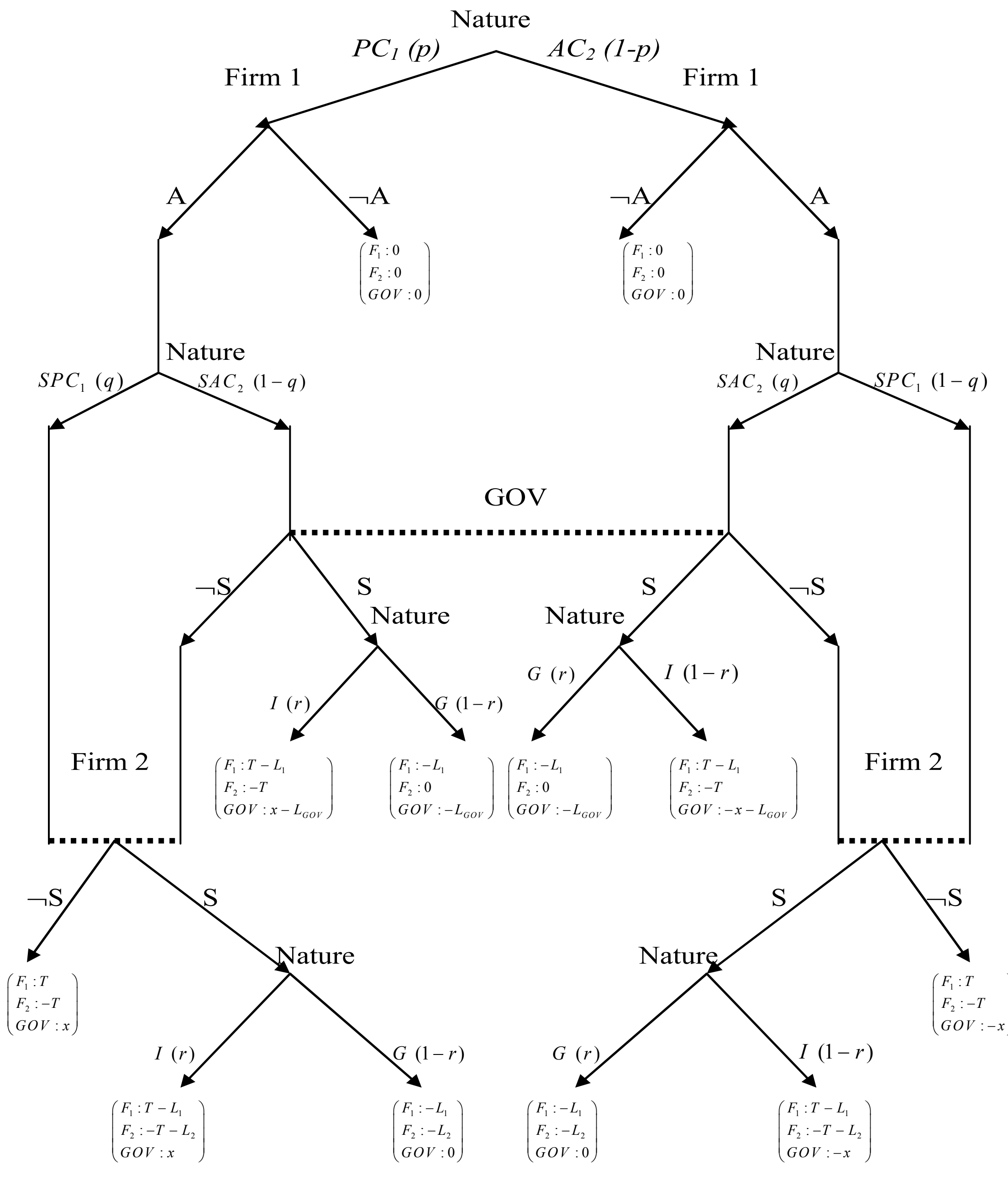
[8] In the U.S., the statute of limitations for private antitrust actions (usually four years) is tolled during the duration of a government antitrust suit. Therefore, firms do not lose anything if they wait for a resolution of the government's case before initiating a private action of their own.

[9] Firm 2 may be able to sue firm 1 even after GOV has already successfully sued firm 1. For simplicity, we focus on the initial suit, which is the most important one and sets the precedent for any possible follow-up suit. Moreover, follow-up suits generally face double jeopardy challenges (see Garoupa and Gomez, 2005, for a detailed economic analysis of double jeopardy). Nonetheless, we consider the potential effects of follow-up suits in the Supplementary Appendix, and discuss the findings in Section X.

[10] We show in the Supplementary Appendix that the qualitative results reported in the paper also hold in cases where it costs more to bring an antitrust suit than to defend against one.

depends negatively on the transfer, T, and its litigation cost, L_2.[11] GOV's payoff depends negatively on its litigation cost, L_{GOV}, and positively on efficiency, x. Players are risk neutral. The extensive form with public and private enforcement is given in figure 1.

Figure 1: The Game Tree with Private and Public Enforcement



[11] We examine the effects of litigation fee-shifting in the Supplementary Appendix. For an economic analysis of fee-shifting, see Kaplow (1993).

This completes the description of the game between GOV, firm 1, and firm 2. We are interested in determining the welfare effects of allowing private antitrust action. To do so, we compare the outcomes of the game in which both firm 2 and GOV can sue firm 1 (private and public enforcement), to those of the reduced games, in which only GOV can sue firm 1 (pure public enforcement) and in which only firm 2 can sue firm 1 (pure private enforcement).

Before proceeding, we note that the model assumes a simple public enforcement technology, in which GOV's only choice is whether to sue. In reality, GOV may have more discretion, and may be able to affect court accuracy through a choice of enforcement effort, which in turn may affect GOV's litigation costs, under the principle that a more accurate enforcement policy requires more resources. We examine an extension of the model that endogenizes GOV's enforcement effort, and thus GOV's litigation costs and court accuracy under GOV, in the Supplementary Appendix, and we discuss the extent to which our results are robust to this extension in Section X. Although some parts of the analysis are affected, most of the results reported in the paper continue to hold.

IV. Pure Public Enforcement

Consider the reduced game where only GOV can sue firm 1. Let σ_G be GOV's suit probability given the signal SAC_2. Let γ_1 be the probability that firm 1 takes the action when it is type PC_1, and γ_2 the probability that firm 1 takes the AC_2 action. We now present the equilibria and welfare values (measured by the sum of players' equilibrium payoffs) of the game with pure public enforcement.

Proposition 1. The proper Nash equilibria of the game with pure public enforcement.

(i) If $T < q(rT + L_1)$ and $\left(r - \frac{p(1-q)}{p(1-q)+(1-p)q}\right)x > L_{GOV}$, then the unique proper equilibrium is $\gamma_1^* = 1$, $\gamma_2^* = \frac{p(1-q)}{(1-p)q}(\frac{(1-r)x+L_{GOV}}{rx-L_{GOV}}) \in (0,1)$, and $\sigma_G^* = \frac{T}{q(rT+L_1)} \in (0,1)$, and equilibrium welfare is $W^* = px - px(\frac{1-q}{q})\left(\frac{(1-r)x+L_{GOV}}{rx-L_{GOV}}\right) - px(\frac{1-q}{q})\left(\frac{L_1 T}{(rx-L_{GOV})(rT+L_1)}\right)$.

(ii) If $T > q(rT + L_1)$ and $\left(r - \frac{p(1-q)}{p(1-q)+(1-p)q}\right)x > L_{GOV}$, then $\gamma_1^* = \gamma_2^* = 1$, $\sigma_G^* = 1$, and $W^* = px - p(1-q)(1-r)x - (1-p)(1-qr)x - (p(1-q) + (1-p)q)(L_1 + L_{GOV})$.

(iii) If $\left(r - \frac{p(1-q)}{p(1-q)+(1-p)q}\right)x < L_{GOV}$, then $\gamma_1^* = \gamma_2^* = 1$, $\sigma_G^* = 0$, and $W^* = px - (1-p)x$.

Proof. Proofs of all propositions are presented in the Mathematical Appendix.

If GOV's litigation costs are so high that it would not want to sue even if it knew that firm 1 always takes the illegal action (region iii), then GOV never sues, and firm 1 always takes the action, whether or not it is legal. If GOV's litigation costs are lower, but firm 1's litigation costs are so low that it would want to take the illegal action even if it knew that GOV always sues (region ii), then firm 1 always takes the action, whether or not it is legal, and GOV always sues. If GOV's litigation costs are not too high and firm 1's litigation costs are not too low (region i), there is a Nash equilibrium in which GOV randomizes between suing and not suing, and firm 1 randomizes between taking and not taking an illegal action.

Region (i) has another equilibrium, in which firm 1 does not take an action, whether or not it is legal, and GOV sues with high probability if firm 1 takes an action. However, this Nash equilibrium is not proper, in that it does not survive reasonable trembles onto out-of-equilibrium strategies (Myerson, 1978, 1997). If GOV sues with high probability, firm 1 may be deterred from taking any action. But suppose firm 1, on rare occasion, mistakenly takes an efficient action; and also on rare occasion, mistakenly takes an illegal action. The first type of mistake is less costly than the second, since GOV sues with high probability and the court is more often right than wrong. Thus, the first type of mistake should be more common, even though both types of mistakes should be rare. If the first type of mistake is sufficiently more common than the second, then if GOV ever observes an action, GOV would be sufficiently more likely to believe that the action was efficient instead of illegal that it would not want to sue with high probability, so that firm 1 would not be deterred from taking an efficient action. Hence, we have the following result:

Corollary 1. If attention is restricted to proper equilibria, firm 1 is never deterred from taking an efficient action under pure public enforcement.

V. Private and Public Enforcement

We now turn to the model in which both GOV and firm 2 can sue. Let α_1 be the probability that firm 1 takes the action when it is type PC_1, α_2 be the probability that firm 1 takes the AC_2 action, σ_{G2} be the probability that GOV sues when it receives the signal SAC_2, β_1 be the probability that firm 2 sues given that the type of firm 1's action is PC_1 and GOV did not sue, and β_2 be the probability that firm 2 sues given that the type is AC_2 and GOV did not sue. The following proposition presents equilibria and welfare values with both private and public enforcement:

Proposition 2. The subgame perfect Nash equilibria of the game with private and public enforcement.

(A) If $L_2 < (1-r)T < L_1 < rT$, then the unique subgame perfect equilibrium is $\alpha_1^* = 1$, $\alpha_2^* = 0$, $\sigma_{G2}^* = 0$, $\beta_1^* = \beta_2^* = 1$, and equilibrium welfare is $W^* = prx - p(L_1 + L_2)$.

(B) If $L_2 < (1-r)T < rT < L_1$, then $\alpha_1^* = \alpha_2^* = 0$ and $W^* = 0$.

(C) If $(1-r)T < L_2 < rT < L_1$ or $(1-r)T < L_2 < L_1 < rT$, then $\alpha_1^* = 1$, $\alpha_2^* = 0$, $\sigma_{G2}^* = 0$, $\beta_1^* = 0$, $\beta_2^* = 1$, and $W^* = px$.

(D) If $L_2 < L_1 < (1-r)T < rT$, $\alpha_1^* = \alpha_2^* = 1$, $\sigma_{G2}^* = 0$, $\beta_1^* = \beta_2^* = 1$, $W^* = px - (1-r)x - (L_1 + L_2)$.

(E) If $(1-r)T < rT < L_2 < L_1$, subgame perfection implies $\beta_1^* = \beta_2^* = 0$ and the equilibrium outcomes are the same as in the game of pure public enforcement (see proposition 1).

If firm 2's litigation costs are higher than its expected benefit from suing when firm 1 takes an illegal action (region E), firm 2 never sues regardless of what it knows about firm 1's action type. Knowing this, GOV acts as if there is no chance of private enforcement. In this case, the full game reduces to the game with pure public enforcement, the solution to which is characterized in proposition 1. If firm 2's litigation costs are not prohibitively high (regions A through D), firm 2 sues at least when firm 1 takes an illegal action. In this case, GOV never sues since it can count on firm 2 to always sue if firm 1 takes an illegal action. GOV knows that it is not perfectly informed and might sue firm 1 for taking an efficient action. Thus it prefers to delegate enforcement to firm 2 whenever it can. GOV only acts if it expects no private enforcement, as is the case if firm 2's litigation costs are too high. Therefore, we have the following result:

Corollary 2. If firm 2's litigation costs are not too high, public enforcement gives way to private enforcement when the two are in play.

If firm 2's litigation costs are lower than its expected benefit from suing when firm 1 takes a legal action (regions A, B, and D), firm 2 always sues, regardless of the legality of firm 1's action. If, instead, firm 2's litigation costs are higher than its expected benefits from wrongfully accusing firm 1, but lower than its expected benefits from rightly accusing firm 1 (region C), as is the case if the court is sufficiently accurate, then firm 2 sues if and only if firm 1 takes an illegal action. In this case, firm 1 need not worry about being sued if it takes the efficient action, and therefore always takes the efficient action. On the other hand, if it takes the illegal action, then it expects that it will be sued by firm 2 with certainty, and therefore does not take the illegal action. Hence, firm 1 only takes the action if it is legal, and firm 2 only sues if firm 1 takes an illegal action.

From proposition 2, we can easily deduce the equilibria of the reduced game with pure private enforcement. In regions (A) through (D), GOV never sues, so the results for the full game with private and public enforcement are the same as those for the reduced game with pure private enforcement. In region (E), firm 2's litigation costs are so high that it never sues. In this region, under pure private enforcement, $\alpha_1^* = \alpha_2^* = 1$, $\beta_1^* = \beta_2^* = 0$, and welfare is $W^* = px - (1-p)x$.

VI. Social Welfare

We now compare the full game with private and public enforcement and the reduced games with pure private enforcement and pure public enforcement in terms of welfare. We assume that, under pure public enforcement, GOV wants to sue if firm 1 always takes the illegal action, and firm 1 does not want to take the illegal action if GOV always sues, that is, we focus on region (i) of parameter space (see proposition 1). This implies that, under private enforcement, firm 1 does not want to take the illegal action if firm 2 always sues, that is, region (D) is also excluded (see proposition 2). We begin by analyzing the welfare effects of adding private enforcement to public enforcement.

Welfare is comprised of various elements: the probability that an illegal action is deterred, the probability that an illegal action is taken but overturned by the court, the probability that a legal action is deterred, the probability that a legal action is overturned, and the expected trial costs. Table 1 summarizes the effects of adding private to public enforcement on these elements of welfare.

We can compare the elements of welfare under private and public, and under pure public, enforcement across the relevant regions of parameter space. With private enforcement, the illegal action is always deterred in all relevant regions. In contrast, under pure public enforcement, the illegal action is never deterred with probability 1. The probability of an illegal action being deterred or overturned is always higher with private enforcement than under pure public enforcement.

Under pure public enforcement, there is no region in which firm 1 is deterred from taking an efficient action (corollary 1). In contrast, with private enforcement, firm 1 is completely deterred from taking a legal action in region (B). In this case, the court is inaccurate enough that firm 2 always sues when firm 1 takes a legal action. Firm 1 thus prefers not to take a legal action, to avoid being wrongfully sued and possibly wrongfully required to pay damages.

Table 1: The Anatomy of Welfare Under Private and Public, and Pure Public Enforcement

Region	Element of Welfare	Private and Public	Pure Public	Comparison
(A)	Illegal action deterred	1 $>$	$1-\gamma_2^*$	$1 > 1-\gamma_2^*(1-q\,\sigma_G^*\,r)$ Private better
	Illegal action overturned	0 $<$	$\gamma_2^*\,q\,\sigma_G^*\,r$	
	Legal action not deterred	1 $=$	1	$r < 1-(1-q)\,\sigma_G^*(1-r)$ Public better
	Legal action not overturned	r $<$	$1-(1-q)\,\sigma_G^*(1-r)$	
	Expected trial costs	$P(L_1+L_2)$ $>=<$	$\sigma_G^*[p(1-q)+(1-p)q\,\gamma_2^*](L_1+L_{Gov})$	Ambiguous
(B)	Illegal action deterred	1 $>$	$1-\gamma_2^*$	$1 > 1-\gamma_2^*(1-q\,\sigma_G^*\,r)$ Private better
	Illegal action overturned	0 $<$	$\gamma_2^*\,q\,\sigma_G^*\,r$	
	Legal action not deterred	0 $<$	1	Public better
	Legal action not overturned	0 $<$	$1-(1-q)\,\sigma_G^*(1-r)$	
	Expected trial costs	0 $<$	$\sigma_G^*[p(1-q)+(1-p)q\,\gamma_2^*](L_1+L_{Gov})$	Private better
(C)	Illegal action deterred	1 $>$	$1-\gamma_2^*$	$1 > 1-\gamma_2^*(1-q\,\sigma_G^*\,r)$ Private better
	Illegal action overturned	0 $<$	$\gamma_2^*\,q\,\sigma_G^*\,r$	
	Legal action not deterred	1 $=$	1	$1 > 1-(1-q)\,\sigma_G^*(1-r)$ Private better
	Legal action not overturned	1 $>$	$1-(1-q)\,\sigma_G^*(1-r)$	
	Expected trial costs	0 $<$	$\sigma_G^*[p(1-q)+(1-p)q\,\gamma_2^*](L_1+L_{Gov})$	Private better

The probability of a legal action being taken and not overturned by the court is higher under pure public enforcement than with private enforcement in all regions except (C), where legal actions are always taken but never sued under private and public enforcement, and always taken but occasionally sued and overturned by the court under pure public enforcement. In the majority of cases, legal actions are taken and survive with higher probability under pure public enforcement.

In region (A), where firm 1 only takes the action if it is efficient and firm 2 always sues firm 1, the effect on the expected trial costs of adding private enforcement to public enforcement is ambiguous. In contrast, in regions (B) and (C), the trial probability is 0 with private enforcement, whereas it is positive under pure public enforcement. Thus, the trial probability is always higher under pure public enforcement in regions (B) and (C).

In region (C), which obtains if the court is accurate enough, the probability of an illegal action is lower, the probability of a legal action is higher, and the expected costs of trial are lower with private enforcement than under pure public enforcement. Thus, private enforcement achieves the overall socially optimal outcome in this region. If the court is accurate enough, the truth about firm 1's action would likely be known if firm 1 were to take the action and be brought to court for it. Firm 2 then always sues if it learns that firm 1 took the illegal action, and never sues if it learns that firm 1 took the legal action. Thus, firm 1 always takes the legal action and never takes the illegal action.

In regions (A) and (B), where the court is not so accurate, adding private to public enforcement increases welfare if and only if GOV's litigation costs are sufficiently high and GOV is sufficiently misinformed (L_{GOV} is high enough and q is low enough). In these regions, firm 2 would sue with positive probability if firm 1 took an efficient action. Society prefers to avoid these inefficiencies by disallowing private suits if GOV has low enough litigation costs and is not too misinformed. Thus we have the following result about the welfare effects of adding private enforcement to public enforcement:

Corollary 3. Private enforcement unambiguously increases welfare if the court is sufficiently accurate; otherwise, it increases welfare only if the government's litigation costs are sufficiently high.

Comparing the efficiency of no enforcement, pure private enforcement, pure public enforcement, and public and private enforcement, we arrive at another result:

Corollary 4. As long as society prefers some enforcement to no enforcement at all, pure private enforcement cannot be strictly optimal.

In regions (A) through (C), GOV never sues under private and public enforcement, so pure private enforcement yields the same level of welfare as private and public enforcement. Moreover, in these regions, pure public enforcement yields higher welfare than private and public enforcement if GOV is sufficiently efficient. In region (E), GOV sues with positive probability, but firm 2 never sues. Thus, in this region, private and public enforcement yields the same level of welfare as pure public enforcement, and these mechanisms result in some public enforcement. Moreover, in this region, pure private enforcement results in no enforcement at all. Thus, as long as society prefers some enforcement to none at all, pure private enforcement cannot be strictly optimal.

If the court is sufficiently accurate, a mechanism with private enforcement achieves the best outcome. But if the court is not so accurate, neither of the mechanisms analyzed so far achieves the best outcome. What mechanisms maximize social welfare in general?

VII. Optimal Public Enforcement: Fee for Public Lawsuit

In the setup so far, firm 2 cannot credibly reveal its full information to GOV before GOV sues because part of its information is not verifiable. For this reason, public enforcement cannot achieve the social optimum. However, one way that firm 2 may be able to signal its information to GOV is through its willingness to pay a fee for public lawsuit. If the amount of the fee is such that firm 2 is only willing to pay the fee if it knows that firm 1's action was illegal, the payment would be fully revealing of firm 2's information. GOV would then be as well informed as firm 2, and public enforcement could achieve the social optimum provided GOV continues to have the correct incentives. To preserve GOV's incentives and avoid any risk of capture, the fee revenue could go to society.[12] We now extend the model to allow signaling through a fee for public lawsuit.

The model is the same as the basic model presented in section III, except that firm 2 has a choice of whether to pay a fee for public action. At time 4, firm 2 costlessly complains to GOV about firm 1's action *and* chooses whether to pay a fee $f > 0$ in the event that GOV chooses to sue firm 1. Then, at time 5, GOV chooses whether to sue firm 1, knowing the signal SAC_2 and whether firm 2 has decided to pay f if GOV chooses to sue. GOV can sue firm 1 even if firm 2 has chosen not to pay f. If GOV chooses not to sue, at time 6, firm 2 chooses whether to sue firm 1 on its own. In this extended framework, the optimum is achieved through public enforcement under certain conditions:

Proposition 3. The social optimum is achieved through a mechanism of public enforcement and a fee f for public lawsuit, in which (1) f satisfies $(1-r)T < f < rT$, (2) public lawsuit is contingent on f being paid, and (3) there is no private enforcement.

In general, the social optimum is achieved through public enforcement with a fee for public action only without private enforcement. Suppose firm 2 can sue on its own. If GOV does not sue, then if firm 2's litigation costs are not too high, firm 2 sues on its own, at least when firm 1 takes an illegal action. Knowing this, GOV has no reason to sue since firm 2 sues whenever firm 1 has taken a illegal action (see corollary 2). Knowing this, firm 2 in turn has no reason to pay the fee for public lawsuit, whether or not the action is legal. Therefore, a fee for public lawsuit cannot achieve the socially optimal outcome if there is private enforcement.

[12] For an analysis of law enforcement with a fee-maximizing government, see Garoupa and Klerman (2002).

Moreover, in general, the optimum is achieved through public enforcement with a fee for public action only if GOV cannot sue unless the fee is paid. If GOV can sue even if firm 2 chooses not to pay the fee, then a *free-riding* problem can arise. If firm 2 does not pay the fee, then GOV's information is the same as in the game without the option of a fee for public suit. In that game, GOV sues with positive probability in some parameter ranges. Thus, firm 2 can expect GOV to sue with positive probability even if it does not pay the fee. Therefore, firm 2 may prefer to free-ride on GOV's lawsuit. In this case, GOV would be no better informed than without a fee for public action.

If there is no private enforcement and GOV can only sue if firm 2 pays the fee f, then f can be set to always induce efficient information revelation. If f is set greater than $(1-r)T$, which is firm 2's expected benefit from a public lawsuit if it knows that firm 1's action was efficient, and smaller than rT, which is firm 2's expected benefit from a public lawsuit if it knows that firm 1's action was illegal, then firm 2 only pays f if it knows that firm 1's action was illegal. In that case, if firm 2 chooses to pay f, GOV knows that firm 1's action was illegal, and if firms 2 chooses not to pay f, it knows that firm 1's action was efficient. Therefore, GOV is perfectly informed, and since it also has the correct incentives, the first best outcome is achieved. That is, firm 1 takes the action only if it is efficient, and GOV sues firm 1 only if firm 1 takes the illegal action.

VIII. Optimal Private Enforcement: Multiplying and Decoupling Damages

In the basic model, private damages are simple, in the sense that a winning plaintiff receives exactly the amount of damages, T, resulting from an action by the defendant. Private damages can be multiplied so that the losing defendant pays a multiple, say $N \geq 1$, of T. We have also assumed that damages are coupled, in the sense that the plaintiff receives what the defendant pays. Private damages can be decoupled so that the plaintiff receives a fraction, say $\delta \geq 0$, of the amount that the defendant pays. It turns out that, in general, a mechanism with private enforcement can achieve the socially optimal outcome if private damages are appropriately multiplied and decoupled.

To prove this, we redefine the mechanisms with private enforcement in terms of the damage multiplier and decoupler. Firm 1 and firm 2's payoffs are the same as before except when the court rules against the defendant. In this case, firm 1 pays NT, and firm 2 receives δNT. The rest of the payment, $(1-\delta)NT$, is assumed to go to society. As before, the damage payment is assumed to be a non-distortionary redistribution between the firms. The parameter regions of the mechanisms with private enforcement are easily redefined in terms of NT and δNT instead of T. Within this more general framework, we obtain the following result about the optimal mechanism:

Proposition 4. The optimal outcome is achieved through a mechanism with private enforcement and a private damage multiplier N and decoupler δ that satisfy (1) $N > \frac{T-L_1}{rT}$ and (2) $\frac{L_2}{rT} < \delta N < \frac{L_2}{(1-r)T}$.

Condition (1) ensures that firm 1 does not take an illegal action if firm 2 would sue firm 1 for taking it. Condition (2) ensures that firm 2 sues firm 1 if and only if firm 1 takes an illegal action. Together these conditions guarantee that firm 1 is always deterred from taking an illegal action and firm 2 never strategically abuses the laws, so that firm 1 is never deterred from taking an efficient action. If these two conditions are satisfied, private enforcement yields the overall social optimum.

IV. Implementability of Optimal Mechanisms

We have identified two mechanisms that are optimal in the model. Public enforcement achieves the optimum with an efficiently set fee for public lawsuit, and private enforcement achieves the optimum with an efficiently set multiplier and decoupler for private damages. From Proposition 4, the optimal multiplier N and decoupler δ for private enforcement must satisfy $N > \frac{T-L_1}{rT}$ and $\frac{L_2}{rT} < \delta N < \frac{L_2}{(1-r)T}$. To set these two instruments optimally requires knowledge of L_1, L_2, r, and T. In contrast, the optimal fee f must satisfy $(1-r)T < f < rT$. To set this instrument optimally requires only knowledge of r and T. Therefore, we have the following result:

Corollary 5. Setting the fee for public suit that makes public enforcement socially optimal is less informationally demanding than setting the multiplier and decoupler that make private enforcement socially optimal.

However, as Proposition 3 indicates, the mechanism of public enforcement with a fee for public lawsuit also has a disadvantage relative to the mechanism of private enforcement with multiplied and decoupled damages:

Corollary 6. Public enforcement with a fee for public lawsuit can only achieve the social optimum without private enforcement, whereas private enforcement with a damage multiplier and decoupler can achieve the optimum with or without public enforcement.

X. Summary and Discussion

We developed a simple strategic model of antitrust violation and law enforcement that yielded several results of possible use to policy-makers. First, under a system of coupled damages, adding private enforcement to public enforcement is always socially beneficial if the court is sufficiently accurate. To the extent that policy-makers trust in the ability of the courts to arrive at the truth, the model suggests that they should permit and encourage private enforcement of the antitrust laws. Second, under a system of coupled damages, if the court is less accurate, adding private enforcement is beneficial only if public enforcement is sufficiently inefficient. To the extent that the court is prone to mistakes and full-time public enforcers are efficient in litigation because of increasing returns to scale, the model suggests that policy-makers should discourage private enforcement of the antitrust laws in a system with coupled damages.

Third, pure private enforcement is never strictly optimal. Adding public to private enforcement cannot harm and may benefit society, even though public enforcers may not be as well informed as private enforcers. Public enforcement just gives way to private enforcement whenever the latter is preferable. Thus, the model suggests that policy-makers should favor the maintenance of a public enforcer of the antitrust laws, assuming of course that it is not prone to malfeasance.

Fourth, the social optimum can be achieved under a system of public enforcement in which firms can pay a fee to initiate government lawsuits but cannot sue on their own, and in which the government can only sue if the fee is paid. In such a system, an appropriately set fee for public lawsuits reveals all private information to the government. Fifth, the social optimum can be achieved under a system of private enforcement with damages that are multiplied and decoupled. Sufficiently multiplying damages ensures that firms do not take illegal actions, while sufficiently decoupling them ensures that firms do not strategically abuse the antitrust laws.

Sixth, knowledge of firms' litigation costs is not required to set the fee that makes public enforcement socially optimal, whereas such knowledge is required to set the damage multiplier and decoupler that make private enforcement optimal. Seventh, private enforcement can achieve the socially optimal outcome in the presence of public enforcement, whereas public enforcement can only achieve the optimum in the absence of private enforcement.

The model we developed in this paper applies to many, but not all, types of antitrust violations. In the model, a firm may be harmed if a rival takes an anticompetitive action, and may privately sue its rival for taking such an action. This is the case for many types of antitrust violations, including predatory pricing, monopoly leveraging, strategic capacity preemption, vertical integration as a threat to competition, exclusive dealing arrangements, product tying arrangements, and other practices to raise rivals' costs or exclude competition. However, in the case of price-fixing arrangements, firms that are not part of the arrangement may benefit from the illegal activity and thus have little incentive to sue. Moreover, if exclusionary practices are successful in eliminating competition, the harmed firms may not be around to sue. Given these possibilities, public enforcement may provide additional benefits that are not captured in the model.

The model also embodies several other important assumptions. First, it assumes that the government's only choice is whether to sue. In reality, it may have more discretion, and may also choose an enforcement effort that may affect court accuracy. In the Supplementary Appendix, we extend the model to endogenize enforcement effort by the government. With more discretion on the part of the government, court accuracy might be different depending on whether the plaintiff is the government or the private firm. Indeed, the government might become better informed at the trial stage than the private plaintiff would. We find that if effort by the government sufficiently improves court accuracy, public enforcement does not always give way to private enforcement when the two are in play. However, all the other qualitative results derived in the paper continue to hold.

Second, the model assumes that, while the private firm may have incentives to sue strategically, the government wants an action to be taken if and only if it is procompetitive. In reality, the government's lawyers may have incentives to win (big) cases independent of their actual welfare consequences. However, the government has many cases from which to choose, and this allows the government's economists to steer attention toward the most socially egregious offenses. Blair and Kaserman (1976) and Kaserman and Mayo (2005) present empirical evidence that the Antitrust Division of the U.S. D.O.J. has tended to bring more cases in those broadly defined industries where the potential for improved consumer welfare was greatest. Nonetheless, some government actors are likely to be partly motivated by factors other than efficiency, including career concerns and political influences. The more the government is motivated by concerns other than efficiency, the less efficient public enforcement can be relative to private enforcement.

Third, the model assumes that, under private enforcement, multiplying private damages does not affect market structure and efficiency. If multiplying damages alters market structure by making convicted firms less viable, and thereby harms social welfare, then optimal private enforcement may be harder to implement. Fourth, the model assumes that the private firm cannot sue its competitor after its competitor has already been successfully sued by the government. In the Supplementary Appendix, we consider the effects of private follow-on suits in the model. Follow-on suits only decrease the probability of public suits under public enforcement, and do not change the other qualitative results in the paper. However, follow-on suits may have another effect that is not captured in the model. Fear of follow-on suits may make it harder for the government to settle out of court with potential or actual antitrust violators. Extending the model to consider the possibility of settlement before trial, and the effects of different enforcement regimes and follow-on suits on the settlement probability, is an interesting avenue for future research.

Mathematical Appendix

Proof of Proposition 1. Consider first GOV's decision. Suppose GOV receives the signal SAC_2. GOV randomizes between S and $\neg S$ if

(1) $rx - Bx - L_{GOV} = 0$,

where $B = \frac{\gamma_1 p(1-q)}{\gamma_1 p(1-q)+\gamma_2(1-p)q}$. GOV chooses S iff $rx - Bx - L_{GOV} > 0$. Consider next firm 1's decision. Suppose firm 1 learns the action is type PC_1. Firm 1 randomizes between A and $\neg A$ when

(2) $T-(1-q)\sigma_G((1-r)T+L_1)=0 \Leftrightarrow \sigma_G = \sigma_{Ga} = \frac{T}{(1-q)((1-r)T+L_1)}$.

Firm 1 chooses A given PC_1 iff $T-(1-q)\sigma_G((1-r)T+L_1)>0$. If firm 1 learns that the action is type AC_2, it randomizes between A and $\neg A$ when

(3) $T-q\sigma_G(rT+L_1)=0 \Leftrightarrow \sigma_G = \sigma_{Gb} = \frac{T}{q(rT+L_1)}$.

Firm 1 chooses A given AC_2 if $T-q\sigma_G(rT+L_1)>0$, and $\neg A$ if $T-q\sigma_G(rT+L_1)<0$.

1.(a). Suppose $T<(1-q)((1-r)T+L_1)<q(rT+L_1)$ and $\left(r-\frac{p(1-q)}{p(1-q)+(1-p)q}\right)x > L_{GOV}$. In equilibrium, $\gamma_2 \neq 1$. If $\gamma_2 = 1$, given that $\left(r-\frac{p(1-q)}{p(1-q)+(1-p)q}\right)x > L_{GOV}$, $\sigma_G = 1$ from (1). If $\sigma_G = 1$, from (3), we get $\gamma_2 = 0$ since $T<q(rT+L_1)$, a contradiction. If $\gamma_1 \neq 0$ in equilibrium, γ_2 cannot be zero. If $\gamma_2 = 0$, $B=1$, and thus, $\sigma_G = 0$. Then, from (3), $\gamma_2 = 1$, a contradiction. Now, $\gamma_1 = 0$ implies that $\sigma_{Gb} < \sigma_{Ga} < \sigma_G \leq 1$ from (2) and (3), and thus, $\gamma_1^* = \gamma_2^* = 0$ and $\sigma_G^* \in (\sigma_{Ga}, 1]$ is a Nash equilibrium. Since $(1-q)((1-r)T+L_1)<T$ and $\sigma_{Ga}<1$, such a $\sigma_G^* \in (\sigma_{Ga}, 1]$ exists. But this equilibrium is not proper. Consider small perturbations $\gamma_1 = \varepsilon_1 > 0$ and $\gamma_2 = \varepsilon_2 > 0$ from the equilibrium strategies $\gamma_1^* = \gamma_2^* = 0$. If GOV observes an action and receives signal SAC_2, its belief that the action is of type PC_1 is then $\mu = \frac{\varepsilon_1(1-q)p}{\varepsilon_1(1-q)p+\varepsilon_2(1-p)q}$. Take $\varepsilon_{1k} = \frac{1}{k}, \varepsilon_{2k} = \frac{1}{k^2}$. These perturbations satisfy the condition $\varepsilon_1 > \varepsilon_2$, which are reasonable since deviating from $\gamma_1 = 0$ is less costly for firm 1 than deviating from $\gamma_2 = 0$.

As $k \to \infty$, $\mu \to 1$. This results in $(r-\mu)x < L_{GOV}$, which leads to $\sigma_G^* = 0$. Then, $\gamma_1^* = \gamma_2^* = 0$ is no longer optimal. Hence, $\gamma_1^* = \gamma_2^* = 0$ is not a proper equilibrium. If $0 < \gamma_2 < 1$ in equilibrium, $\sigma_G = \frac{T}{q(rT+L_1)} \in (0,1)$ from (3). For such a σ_G, $T-(1-q)\sigma_G((1-r)T+L_1) > 0$ from (2), which implies $\gamma_1^* = 1$. Then, we get $\gamma_2^* = \frac{p(1-q)}{(1-p)q}(\frac{(1-r)x+L_{GOV}}{rx-L_{GOV}}) \in (0,1)$ from (1). Thus, $\gamma_1^* = 1$, $\gamma_2^* = \frac{p(1-q)}{(1-p)q}(\frac{(1-r)x+L_{GOV}}{rx-L_{GOV}})$, $\sigma_G^* = \frac{T}{q(rT+L_1)}$ is a Nash equilibrium.

1.(b) Suppose $(1-q)((1-r)T+L_1) < T < q(rT+L_1)$ and $\left(r - \frac{p(1-q)}{p(1-q)+(1-p)q}\right)x > L_{GOV}$. As in 1.(a), $\gamma_2 \neq 1$ in equilibrium. In equilibrium, γ_2 cannot be zero either. The condition $(1-q)((1-r)T+L_1) < T$ implies $\gamma_1 = 1$ even if $\sigma_G = 1$. For $\gamma_1 = 1$, if $\gamma_2 = 0$, (1) is always negative, so $\sigma_G = 0$. Then, from (3), we get that $\gamma_2 = 1$, which is a contradiction. For $\gamma_2 \in (0,1)$, equation (3) is binding and we get $\sigma_G^* = \frac{T}{q(rT+L_1)} \in (0,1)$. Then, for such a σ_G, $\gamma_1^* = 1$. From (1), $\gamma_2^* = \frac{p(1-q)}{(1-p)q}(\frac{(1-r)x+L_{GOV}}{rx-L_{GOV}})$. Thus, combining the results from 1.(a) and 1.(b), if $T < q(rT+L_1)$ and $\left(r - \frac{p(1-q)}{p(1-q)+(1-p)q}\right)x > L_{GOV}$, the only proper equilibrium is $\gamma_1^* = 1$, $\gamma_2^* = \frac{p(1-q)}{(1-p)q}(\frac{(1-r)x+L_{GOV}}{rx-L_{GOV}}) \in (0,1)$, and $\sigma_G^* = \frac{T}{q(rT+L_1)} \in (0,1)$.

2. Suppose $T > q(rT+L_1)$ and $\left(r - \frac{p(1-q)}{p(1-q)+(1-p)q}\right)x > L_{GOV}$. The condition $T > q(rT+L_1)$ implies that $\gamma_2 = 1$ even if $\sigma_G = 1$. Since $T > q(rT+L_1) > (1-q)((1-r)T+L_1)$, $\gamma_1 = \gamma_2 = 1$. Then, $\sigma_G = 1$ since $\left(r - \frac{p(1-q)}{p(1-q)+(1-p)q}\right)x > L_{GOV}$. Hence, the unique equilibrium is $\gamma_1^* = \gamma_2^* = 1$ and $\sigma_G^* = 1$.

3. Suppose $\left(r - \frac{p(1-q)}{p(1-q)+(1-p)q}\right)x < L_{GOV}$. As in 1.(a), $\gamma_1^* = \gamma_2^* = 0$ and $\sigma_G^* \in (\sigma_{Ga}, 1]$ is not a proper equilibrium. In equilibrium, it cannot be true that $\gamma_1 \neq 0$ and $\gamma_2 = 0$. If $\gamma_1 \neq 0$ and $\gamma_2 = 0$, $\sigma_G = 0$, which leads to $\gamma_2 = 1$. This is a contradiction. Nor is $\gamma_1 \neq 0$ and $\gamma_2 \in (0,1)$ part of an equilibrium either. If $\gamma_2 \in (0,1)$, $\gamma_1 = 1$ and $\sigma_G = \frac{T}{q(rT+L_1)} \in (0,1)$. But for GOV to randomize, it must be that $\left(r - \frac{p(1-q)}{p(1-q)+\gamma_2(1-p)q}\right)x = L_{GOV}$, which is a contradiction since $\left(r - \frac{p(1-q)}{p(1-q)+\gamma_2(1-p)q}\right)x < \left(r - \frac{p(1-q)}{p(1-q)+(1-p)q}\right)x < L_{GOV}$ in the current parameter range. Thus, the mixed strategies cannot be an equilibrium. If $\gamma_2 = 1$, it implies that $T - q\sigma_G(rT+L_1) > 0$ and $\gamma_1 = 1$. Since the condition $\left(r - \frac{p(1-q)}{p(1-q)+(1-p)q}\right)x < L_{GOV}$ implies that $\sigma_G = 0$ when $\gamma_1 = \gamma_2 = 1$, the unique equilibrium is $\gamma_1^* = \gamma_2^* = 1$ and $\sigma_G^* = 0$. *Q.E.D.*

Proof of Proposition 2. Consider firm 2's decision in the two subgames. If the action type is PC_1, firm 2 chooses S iff $((1-r)T - L_2) > 0$. Similarly, if the type is AC_2, firm 2 sues iff $rT - L_2 > 0$. Now, consider GOV's decision. Given SAC_2, GOV randomizes between S and $\neg S$ when

(4) $\quad -D(1-r)x(1-\beta_1) + (1-D)rx(1-\beta_2) - L_{GOV} = 0$,

where $D = \frac{\alpha_1 p(1-q)}{\alpha_1 p(1-q) + \alpha_2 (1-p) q}$. GOV chooses S iff $-D(1-r)x(1-\beta_1) + (1-D)rx(1-\beta_2) - L_{GOV} > 0$. Consider next firm 1's decision. Consider first the case where firm 1 learns the type is PC_1. Given β_1, β_2, and σ_{G2}, firm 1 randomizes between A and $\neg A$ when

(5) $\quad \left(q + (1-q)(1-\sigma_{G2})\right)\left[\beta_1(rT - L_1) + (1-\beta_1)T\right] + (1-q)\sigma_{G2}(rT - L_1) = 0$.

Similarly, in the case where firm 1 learns the type is AC_2, it randomizes between A and $\neg A$ when

(6) $\quad \left(1 - q + q(1-\sigma_{G2})\right)\left[\beta_2((1-r)T - L_1) + (1-\beta_2)T\right] + q\sigma_{G2}((1-r)T - L_1) = 0$.

A. Suppose $L_2 < (1-r)T < L_1 < rT$. (5) is always positive, so $\alpha_1^* = 1$. Since $L_2 < (1-r)T < rT$, $\beta_1^* = 1$ and $\beta_2^* = 1$. Knowing this, GOV chooses $\neg S$, i.e., $\sigma_{G2}^* = 0$. For $\beta_1^* = 1$, $\beta_2^* = 1$, and $\sigma_{G2}^* = 0$, (6) is always negative, so $\alpha_2^* = 0$. Thus, the SPE is $\alpha_1^* = 1$, $\alpha_2^* = 0$, $\beta_1^* = \beta_2^* = 1$, $\sigma_{G2}^* = 0$.

B. Suppose $L_2 < (1-r)T < rT < L_1$. Since $L_2 < (1-r)T < rT$, we have $\beta_1^* = \beta_2^* = 1$. From (4), $D(-L_{GOV}) + (1-D)(-L_{GOV}) < 0$, so $\sigma_{G2}^* = 0$. Since $\beta_1^* = \beta_2^* = 1$ and $\sigma_{G2}^* = 0$, from (5) and (6), $(rT - L_1) < 0$ and $(1-r)T - L_1 < 0$, so $\alpha_1^* = \alpha_2^* = 0$. The SPE is $\alpha_1^* = \alpha_2^* = 0$, $\beta_1^* = \beta_2^* = 1$, $\sigma_{G2}^* = 0$.

C. Suppose $(1-r)T < L_2 < rT < L_1$ or $(1-r)T < L_2 < L_1 < rT$. Since $(1-r)T < L_2 < rT$, $\beta_1^* = 0$ and $\beta_2^* = 1$. From (4), $D(-L_{GOV} - (1-r)x) + (1-D)(-L_{GOV}) < 0$, so $\sigma_{G2}^* = 0$. For $\beta_1^* = 0$, $\beta_2^* = 1$, and $\sigma_{G2}^* = 0$, (5) is positive but (6) is negative. The SPE is $\alpha_1^* = 1$, $\alpha_2^* = 0$, $\beta_1^* = 0$, $\beta_2^* = 1$, and $\sigma_{G2}^* = 0$.

D. Suppose $L_2 < L_1 < (1-r)T < rT$. Then, (5) and (6) are positive for any β_1, β_2, and σ_{G2}, so $\alpha_1^* = \alpha_2^* = 1$. Since $L_2 < (1-r)T < rT$, $\beta_1^* = \beta_2^* = 1$. Then GOV's payoff is $-L_{GOV} < 0$, so $\sigma_{G2}^* = 0$.

E. Suppose $(1-r)T < rT < L_2 < L_1$. Since $(1-r)T < rT < L_2$, $\beta_1^* = \beta_2^* = 0$, so the game reduces to the game of pure public enforcement (see the proof of proposition 1). *Q.E.D.*

Proof of Proposition 3. To be realistic, we assume that $f < L_{GOV}$. Let λ_1 be the probability that firm 2 pays the fee f for a public lawsuit if the type is PC_1. Similarly, let $\lambda_2 = prob[pay\ f \mid AC_2]$. Let

σ_{G1} be the probability that GOV chooses S if f is paid, and $\sigma_{G2} = prob[S \mid f \neg paid]$. Let $\tilde{\alpha}_1$ be the probability that firm 1 takes a type PC_1 action, and $\tilde{\alpha}_2 = prob[A \mid AC_2]$. Firm 2 never pays f if $f > L_2$. For $f < L_2$, consider first the subgames for firm 2 that follow a decision by GOV to choose $\neg S$, both in the case where firm 2 paid f and in the case where firm 2 did not pay f. Let $\tilde{\beta}_1$ be the probability that firm 2 chooses S if firm 1 takes the type PC_1 action and GOV chooses $\neg S$, and $\tilde{\beta}_2$ be the probability that firm 2 chooses S if firm 1 takes the type AC_2 action and GOV chooses $\neg S$ (irrespective of firm 2's fee payment choices). In these subgames, when the type is PC_1, firm 2 chooses S iff $T(1-r) > L_2$, and when the type is AC_2, firm 2 chooses S iff $rT > L_2$.

Since public enforcement achieves the optimum when firm 2 truthfully reveals its private information to GOV, we focus on separating equilibria. In regions (A), (B), (C), and (D), firm 2 chooses S at least when the action is type AC_2. Knowing this, GOV chooses $\neg S$, so the option of paying f does not affect equilibrium. In general, therefore, optimal public enforcement can only be achieved when there is no private enforcement. For given λ_1 and λ_2, GOV's net expected utility from S when f is paid, is $-L_{GOV} - \tilde{D}_1 x + rx$, where $\tilde{D}_1 = \frac{\lambda_1 \tilde{\alpha}_1 p(1-q)}{\lambda_1 \tilde{\alpha}_1 p(1-q) + (1-p) q \tilde{\alpha}_2 \lambda_2}$. GOV randomizes between S and $\neg S$ iff $\tilde{D}_1 = \frac{rx - L_{GOV}}{x}$. Similarly, GOV's net expected utility from S when f is not paid is $-L_{GOV} - \tilde{D}_2 x + rx$, where $\tilde{D}_2 = \frac{(1-\lambda_1)\tilde{\alpha}_1 p(1-q)}{(1-\lambda_1)\tilde{\alpha}_1 p(1-q) + (1-p) q \tilde{\alpha}_2 (1-\lambda_2)}$. GOV randomizes between S and $\neg S$ iff $D_2 = \frac{rx - L_{GOV}}{x}$. If the type is PC_1, for a fee f satisfying $(1-r)T < f < rT$, firm 2 is always better off not paying f since $\sigma_{G1}\left(f - (1-r)T\right) + \sigma_{G2}(1-r)T > 0$ for all $\sigma_{G1} \in (0,1]$ and $\sigma_{G2} \in (0,1]$. Thus, $\lambda_1^* = 0$. When $\lambda_1^* = 0$, $\tilde{D}_1 = 0$, and thus, GOV always chooses S when f is paid, $\sigma_{G1}^* = 1$. Knowing that $\lambda_1^* = 0$ and $\sigma_{G1}^* = 1$, firm 1 takes a type PC_1 action iff $T > (1-q)\sigma_{G2}((1-r)T + L_1)$. If the type is AC_2, firm 2's net expected payoff from paying f is $(\sigma_{G1} - \sigma_{G2}) rT - \sigma_{G1} f$. Firm 2 randomizes between paying and not paying f iff $(1-\sigma_{G2}) = \frac{f}{rT} \Leftrightarrow \sigma_{G2}^* = \frac{rT - f}{rT}$. Firm 1 randomizes between taking and not taking a type AC_2 iff $(\lambda_2 + \sigma_{G2} - \lambda_2 \sigma_{G2}) = \frac{T}{q(rT + L_1)}$. So $\lambda_2^* = \frac{rT^2}{fq(rT + L_1)} - \frac{(rT - f)}{f}$ and $\tilde{\alpha}_2^* = \frac{p(1-q)}{(1-p)q} \frac{((1-r)x + L_G)}{x} \frac{f}{rT} \frac{q(rT + L_1)}{(q(rT + L_1) - T)}$. That is, when firm 2 randomizes between payment and no payment of f when the type is AC_2, $\lambda_2^* \in (0,1)$, GOV also randomizes with probability $\sigma_{G2}^* = \frac{rT - f}{rT}$ when f is not paid. In this separating equilibrium, $\lambda_1^* = 0$, $\lambda_2^* = \frac{rT^2}{fq(rT + L_1)} - \frac{(rT - f)}{f}$, $\sigma_{G1}^* = 1$, $\sigma_{G2}^* = \frac{rT - f}{rT}$,

$\tilde{\alpha}_2^* = \frac{p(1-q)}{(1-p)q}\frac{((1-r)x+L_G)}{x}\frac{f}{rT}\frac{q(rT+L_1)}{(q(rT+L_1)-T)}$, $\tilde{\alpha}_1^* = 1$ given that $T > (1-q)\frac{(rT-f)((1-r)T+L_1)}{rT}$, and $\tilde{\beta}_1 = \tilde{\beta}_2 = 0$. The outcome is suboptimal. Another separating equilibrium exists when $\sigma_{G2}^* = 0$. In this case, firm 2's net expected payoff from paying f is $\sigma_{G1}(rT - f) > 0$ for any $\sigma_{G1} \in (0,1]$. Thus, $\lambda_2^* = 1$. If GOV always chooses $\neg S$ when f is not paid, firm 2 pays f only if the type is AC_2. Knowing this, firm 1 never takes a type AC_2 action and only takes a type PC_1 action. That is, if $\sigma_{G2}^* = 0$, the separating equilibrium with the fee for public lawsuit is perfectly revealing of the type, so public enforcement can achieve the optimum. In this separating equilibrium, $\lambda_1^* = 0$, $\lambda_2^* = 1$, $\sigma_{G1}^* = 1$, $\sigma_{G2}^* = 0$, $\tilde{\alpha}_2^* = 0$, $\tilde{\alpha}_1^* = 1$, and $\tilde{\beta}_1 = \tilde{\beta}_2 = 0$. In summary, there is a separating equilibrium that implements the socially optimal outcome and this equilibrium is attained if GOV is not allowed to sue unless f is paid by firm 2 and f satisfies $(1-r)T < f < rT$. *Q.E.D.*

Proof of Proposition 4. Consider the mechanism with public and private enforcement. When firm 1 takes a PC_1 action and GOV chooses $\neg S$, firm 2 chooses S iff $(1-r)\delta NT > L_1$. If firm 1 takes an AC_2 action and GOV chooses $\neg S$, firm 2 chooses S iff $r\delta NT > L_1$. GOV chooses S iff $-D(1-r)\left[x(1-\beta_1) + \beta_1(1-\delta)NT\right] + (1-D)r\left[x(1-\beta_2) - \beta_2(1-\delta)NT\right] - L_{GOV} > 0$. Firm 1 takes a PC_1 action iff $\left(q + (1-q)\sigma_{G2}\right)\left[T - \beta_1 L_1 - \beta_1(1-r)NT\right] + (1-q)\sigma_{G2}(rT - L_1) > 0$. Firm 1 takes an AC_2 action iff $\left(q(1-\sigma_{G2}) + (1-q)\right)\left[T - \beta_2 L_1 - \beta_2 rNT\right] + q\sigma_{G2}((1-r)T - L_1) > 0$. Suppose $L_2 < (1-r)\delta NT < r\delta NT$. In this case, $\beta_1^* = \beta_2^* = 1$, and GOV never sues, i.e., $\sigma_{G2}^* = 0$. Regardless of firm 1's action, the outcome is suboptimal. Even if firm 1 always takes a PC_1 action and never takes an AC_2 action, $\alpha_1^* = 1$ and $\alpha_2^* = 0$, the PC_1 action is sometimes overturned since firm 2 always sues even when firm 1 takes a PC_1 action. Suppose $(1-r)\delta NT < L_2 < r\delta NT$. In this range, $\beta_1^* = 0$ and $\beta_2^* = 1$, so $\sigma_{G2}^* = 0$. Thus, $\alpha_1^* = 1$. Moreover, $\alpha_2^* = 0$ if $T - L_1 - rNT < 0$. Hence, the optimal outcome obtains in this range if $T - L_1 - rNT < 0$. Suppose $(1-r)\delta NT < r\delta NT < L_2$. Then $\beta_1^* = \beta_2^* = 0$, so the mechanism of private and public enforcement reduces to that of pure public enforcement, in which N and δ play no role. The outcome is suboptimal in this range since pure public enforcement never yields the optimal outcome, $\alpha_1^* = 1, \alpha_2^* = 0$ (see proposition 1). Hence, the optimal outcome is attained only with private enforcement, and only with N and δ that satisfy $N > \frac{T-L_1}{rT}$ and $\frac{L_2}{(1-r)T} > \delta N > \frac{L_2}{rT}$. *Q.E.D.*

References

Baumol, William J. and Ordover, Janusz A., 1985. "Use of Antitrust to Subvert Competition," *Journal of Law and Economics* 28, 247-265.

Becker, Gary S. and Stigler, George J., 1974. "Law Enforcement, Malfeasance and Compensation of Enforcers," *Journal of Legal Studies* 3, 1-18.

Besanko, David and Spulber, Daniel F., 1990. "Are Treble Damages Neutral? Sequential Equilibrium and Private Antitrust Enforcement," *American Economic Review* 80, 870-887.

Blair, Roger D. and Kaserman, David L., 1976. "Market Structure and Costs: An Explanation of the Behavior of the Antitrust Authorities," *Antitrust Bulletin* 21, 691-702.

Block, Michael K., Nold, F.C., and Sidak, J. Gregory, 1981. "The Deterrent Effect of Antitrust Enforcement," *Journal of Political Economy* 89, 429-445.

Breit, William and Elzinga, Kenneth G., 1985. "Private Antitrust Enforcement: The New Learning," *Journal of Law and Economics* 28, 405-443.

Briggs, Hugh C. III, Huryn, Kathleen D., and McBride, Mark E., 1996. "Treble Damages and the Incentive to Sue and Settle," *RAND Journal of Economics* 27, 770-786.

Brodley, Joseph F., 1995. "Antitrust Standing in Private Merger Cases: Reconciling Private Incentives and Public Enforcement Goals," *Michigan Law Review* 94, 1-108.

Easterbrook, Frank H., 1985. "Detrebling Antitrust Damages," *Journal of Law and Economics* 28, 445-467.

European Commission, 2005. "Green Paper on Damages Actions for Breach of the EC Antitrust Rules," COM(2005) 672 final, 19 Dec 2005.

Garoupa, Nuno, 1997. "A Note on Private Enforcement and Type I Error," *International Review of Law and Economics* 17, 423-429.

Garoupa, Nuno and Klerman, Daniel, 2002. "Optimal Law Enforcement with a Rent-Seeking Government," *American Law and Economics Review* 4, 116-140.

Garoupa, Nuno and Gomez-Pomar, Fernado, 2005. "Punish Once or Punish Twice: A Theory of the Use of Criminal Sanctions in Addition to Regulatory Penalties," *American Law and Economics Review* 6, 410-433.

Gentry, William M. and Hubbard, R. Glenn, 2000. "Tax Policy and Entrepreneurial Entry," *American Economic Review* 90, 283-287.

Ginsburg, Douglas H., 2005. "Comparing Antitrust Enforcement in the United States and Europe," *Journal of Competition Law and Economics* 1, 427-439.

Hadfield, Gillian K., 2002. "Privatizing Commercial Law: Lessons from ICANN," *Journal of Small and Emerging Business Law* 6, 257-288.

Kaplow, Louis, 1993. "Shifting Plaintiffs' Fees Versus Increasing Damage Awards," *RAND Journal of Economics* 24, 625-630.

Kaserman, David L. and Mayo, John W., 1995. *Government and Business: The Economics of Antitrust and Regulation*, The Dryden Press.

Landes, William M. and Posner, Richard A., 1975. "The Private Enforcement of Law," *Journal of Legal Studies* 4, 1-46.

McAfee, R. Preston, Mialon, Hugo M., and Mialon, Sue H., 2006. "Private Antitrust Litigation: Pro-competitive or Anti-competitive?", in *The Political Economy of Antitrust*, Ghosal and Stennek eds., North-Holland.

McAfee, R. Preston and Vakkur, Nicholas, 2004. "The Strategic Abuse of Antitrust Laws," *Journal of Strategic Management Education* 1, 1-18.

Myerson, Roger B., 1978. "Proper Equilibria," *International Journal of Game Theory* 7, 73-80.

Myerson, Roger B., 1997. *Game Theory: Analysis of Conflict*, Cambridge: Harvard University Press.

Newmark, Craig M., 1988. "Is Antitrust Enforcement Effective?" *Journal of Political Economy* 96, 1315-28.

Polinsky, A. Mitchell, 1980. "Private versus Public Enforcement of Fines," *Journal of Legal Studies* 9, 105-127.

Polinsky, A. Mitchell and Che, Yeon-Koo, 1991. "Decoupling Liability: Optimal Incentives for Care and Litigation," *RAND Journal of Economics* 22, 562-570.

Posner, Richard A., 1992. *Economic Analysis of Law*, 4th ed., Boston: Little Brown.

Roach, Kent and Trebilcock, Michael J., 1996. "Private Enforcement of Competition Laws," *Osgoode Hall Law Journal* 34, 461-508.

Salant, Stephen W., 1987. "Treble Damage Awards in Private Lawsuits for Price Fixing," *Journal of Political Economy* 95, 1326-1336.

Schwartz, Warren F., 1980. "An Overview of the Economics of Antitrust Litigation," *Georgetown Law Journal* 68, 1075-1102.

Segal, Ilya and Whinston, Michael D., 2006. "Public vs. Private Enforcement of the Antitrust Laws: A Survey," Stanford Law and Economics Olin Working Paper No. 335 Available at SSRN: http://ssrn.com/abstract=952067.

Shavell, Steven M., 1984. "Liability for Harm versus Regulation of Safety," *Journal of Legal Studies* 13, 357-374.

Shavell, Steven M., 1997. "The Fundamental Divergence Between the Private and Social Motive to Use the Legal System," *Journal of Legal Studies* 26, 575-612.

Shughart II, William F., 1990. "Private Antitrust Enforcement: Compensation, Deterrence, or Extortion?", *Regulation* 13, 53-61.

Wils, Wouter P.J., 2003. "Should Private Antitrust Enforcement Be Encouraged in Europe?" *World Competition* 26, 473-488.

Supplementary Appendix

In this appendix, we consider additional extensions of the model, and show that the main qualitative results derived in the paper continue to hold.

Fee Shifting. Under the "English" fee-shifting rule, the party that loses in court pays the other party's attorney's fees. That is, if the plaintiff (firm 2) wins, the defendant (firm 1) pays the litigation costs of the plaintiff (L_2), and vice versa. This rule is applied only in the case of private enforcement. Thus, Proposition 1, which is about the case of public enforcement, is not affected.

First, consider the effect of fee shifting in the case of single and coupled private damages. Let σ_{G2f} be the probability that GOV chooses S if it receives the signal SAC_2. Let α_{1f} be the probability that firm 1 takes the type PC_1 action. Similarly, let $\alpha_{2f} = prob[A \mid AC_2]$, $\beta_{1f} = prob[S \mid PC_1, \sigma_{G2f} = 0]$, and $\beta_{2f} = prob[S \mid AC_2, \sigma_{G2f} = 0]$. When the type is PC_1, firm 2 chooses S iff $(1-r)T - r(L_1 + L_2) > 0$. Similarly, when the type is AC_2, firm 2 sues iff $rT - (1-r)(L_1 + L_2) > 0$. Given SAC_2, GOV's net expected utility from S is

(7) $$-D_f(1-r)x(1-\beta_{1f}) + (1-D_f)rx(1-\beta_{2f}) - L_{GOV},$$

where $D_f = \frac{\alpha_{1f}p(1-q)}{\alpha_{1f}p(1-q)+\alpha_{2f}(1-p)q}$. GOV chooses S if (7) is positive, $\neg S$ if (7) is negative, and randomizes if (7) equals zero. When $(\frac{1-r}{r})T < L_1 + L_2 < (\frac{r}{1-r})T$ or $L_1 + L_2 < (\frac{1-r}{r})T < (\frac{r}{1-r})T$, $\beta_{2f} = 1$. Knowing this, GOV always chooses $\neg S$, i.e., $\sigma^*_{G2f} = 0$. But if $(\frac{1-r}{r})T < (\frac{r}{1-r})T < L_1 + L_2$, firm 2 never sues, i.e., $\beta_{1f} = \beta_{2f} = 0$. Knowing this, GOV chooses S with a probability $\sigma^*_{G2} \in [0,1]$. Given β_{1f}, β_{2f}, and σ_{G2f}, if firm 1 learns that the type is PC_1, firm 1 randomizes between A and $\neg A$ when

(8) $$\left(q + (1-q)(1-\sigma_{G2f})\right)\left[\beta_{1f}(rT - (1-r)(L_1 + L_2)) + (1-\beta_{1f})T\right] + (1-q)\sigma_{G2f}(rT - L_1) = 0.$$

Similarly, in the case where firm 1 learns the type is AC_2, it randomizes between A and $\neg A$ when

(9) $\quad \left(1-q+q(1-\sigma_{G2f})\right)\left[\beta_{2f}((1-r)T-r(L_1+L_2))+(1-\beta_{2f})T\right]+q\sigma_{G2f}((1-r)T-L_1)=0$.

F. Suppose $L_1+L_2<(\frac{1-r}{r})T<(\frac{r}{1-r})T$. Then, $\beta^*_{1f}=1$, $\beta^*_{2f}=1$. Knowing this, GOV chooses $\neg S$, i.e., $\sigma^*_{G2f}=0$. For $\beta^*_{1f}=1$, $\beta^*_{2f}=1$, and $\sigma^*_{G2f}=0$, (8) and (9) are positive, so $\alpha^*_{1f}=1$ $\alpha^*_{2f}=1$. Therefore, the SPE is $\alpha^*_{1f}=1$ $\alpha^*_{2f}=1$, $\beta^*_{1f}=1$, $\beta^*_{2f}=1$, and $\sigma^*_{G2f}=0$.

G. Suppose $(\frac{1-r}{r})T<L_1+L_2<(\frac{r}{1-r})T$. Then, $\beta^*_{1f}=0$ and $\beta^*_{2f}=1$. Then, from (7), $D_f(-L_{GOV}-(1-r)x)+(1-D_f)(-L_{GOV})<0$, so $\sigma^*_{G2f}=0$. Then, for $\beta^*_{1f}=0, \beta^*_{2f}=1$, and $\sigma^*_{G2f}=0$, (8) is positive, but (9) is negative. Thus, the unique SPE is $\alpha^*_{1f}=1$ $\alpha^*_{2f}=0$, $\beta^*_{1f}=0, \beta^*_{2f}=1$, and $\sigma^*_{G2f}=0$.

H. Suppose $(\frac{1-r}{r})T<(\frac{r}{1-r})T<L_1+L_2$. Then, $\beta^*_{1f}=\beta^*_{2f}=0$, so the game reduces to the game of pure public enforcement where fee shifting does not affect the GOV's problem or firm 1's problem. When $\beta^*_{1f}=\beta^*_{2f}=0$, (7) reduces to $-D_f(1-r)x+(1-D_f)rx-L_{GOV}$, which is equivalent to the case of pure public enforcement without fee shifting. (see the proof of proposition 1).

Fee shifting only affects the parameter ranges wherein the different equilibria occur, but not the configuration of equilibria. The equilibrium in region F (G, H) under fee shifting is equivalent to the equilibrium in region D (C, E) under no fee shifting. In region F, firm 1 is never deterred from taking an anti-competitive action, and firm 2 sues strategically. Thus, private enforcement does not always improve welfare, but it improves welfare if r is sufficiently high, which leads to the same conclusion as Corollary 3. In region H, in the case of pure private enforcement, there are no antitrust suits but there are antitrust violations, so public suits are needed. Thus, Corollary 4 continues to hold. In regions F through H, public enforcement gives way to private enforcement when the two are in play. Hence, Corollary 2 also continues to hold.

Allowing private damages to be multiplied and decoupled in the model with fee-shifting, we find that private enforcement achieves the socially optimal outcome with a damage multiplier N and decoupler δ satisfying $(1)_{\text{fs}}$ $N>\frac{T-r(L_1+L_2)}{rT}$ and $(2)_{\text{fs}}$ $\frac{(1-r)(L_1+L_2)}{rT}<\delta N<\frac{r(L_1+L_2)}{(1-r)T}$. Condition $(2)_{\text{fs}}$ ensures that firm 2 never sues strategically, and $(1)_{\text{fs}}$ ensures that firm 1 never takes an anti-competitive action, in the model with fee-shifting. Thus, we obtain a result similar to the result in Proposition 4. Under fee-shifting the optimal multiplier can be lower because fee-shifting increases deterrence. However, for a given multiplier, the optimal decoupler under fee-shifting can be higher or lower than under no fee-shifting depending on the values of the other parameters.

Higher Litigation Costs for the Plaintiff. Assuming that $L_1 < L_2$ instead of $L_1 > L_2$ does not affect the results about pure and optimal public enforcement, so Propositions 1 and 3 are unaffected. In the case of private enforcement, instead of the five parameter ranges specified in Proposition 2, we have six parameter ranges, only one of which has an equilibrium different from any in Proposition 2:

Region C-1: $(1-r)T < L_1 < L_2 < rT$. Since $(1-r)T < L_2 < rT$, $\beta_1^* = 0$ and $\beta_2^* = 1$. Then, from (4), $\sigma_{G2}^* = 0$. Then, for $\beta_1^* = 0$, $\beta_2^* = 1$, and $\sigma_{G2}^* = 0$, (5) is positive, but (6) is negative. Thus, the unique SPE is $\alpha_1^* = 1$, $\alpha_2^* = 0$, $\beta_1^* = 0$, $\beta_2^* = 1$, and $\sigma_{G2}^* = 0$, which is the same as the equilibrium in region C in the model where $L_1 > L_2$. Region D-1: $L_1 < L_2 < (1-r)T < rT$. Since $L_2 < (1-r)T < rT$, $\beta_1^* = \beta_2^* = 1$. Then GOV's payoff is $-L_{GOV} < 0$, so $\sigma_{G2}^* = 0$ .Then, (5) and (6) are positive for any β_1 , β_2 , and σ_{G2} , so $\alpha_1^* = \alpha_2^* = 1$, which is the same as the equilibrium in region D in Proposition 2. Regions E-1, E.2, and E.3: $(1-r)T < rT < L_1 < L_2$, $(1-r)T < L_1 < rT < L_2$, and $L_1 < (1-r)T < rT < L_2$. Since $(1-r)T < rT < L_2$, $\beta_1^* = \beta_2^* = 0$, so the game reduces to the game of pure public enforcement, as in region E in Proposition 2. Region I: $L_1 < (1-r)T < L_2 < rT$. Since $(1-r)T < L_2 < rT$, $\beta_1^* = 0$ and $\beta_2^* = 1$. Then, from (4), $\sigma_{G2}^* = 0$. Then, for $\beta_1^* = 0$, $\beta_2^* = 1$, and $\sigma_{G2}^* = 0$, both (5) and (6) are positive. Thus, the unique SPE is $\alpha_1^* = 1$, $\alpha_2^* = 1$, $\beta_1^* = 0$, $\beta_2^* = 1$, and $\sigma_{G2}^* = 0$.

The only qualitative difference in equilibrium occurs in region I if we assume $L_1 < L_2$ instead of $L_1 > L_2$. In this region, there are no strategic suits, but anticompetitive actions are not deterred. Therefore, the only difference if we assume $L_1 < L_2$ is that under private enforcement strategic abuse is less of a problem, but deterrence of illegal actions is a greater problem. Aside from this, there is no change in the qualitative results in Corollaries 2 through 6 and Propositions 3 and 4.

Follow-up Suits. We model follow-up suits in the simplest way. Suppose that a successful public suit always leads to a follow-up suit by firm 2, which is always successful as well given the public precedent. In this case, when firm 1 looses a public suit, it pays T in the public suit, and then pays an additional ϕT to firm 2 in the follow-up suit. A successful public suit restores the market to the state it would have been in had firm 1 not taken the action, and the follow-up suit simply multiplies the damages that firm 1 has to pay if it looses a public suit.

Let α_{1p} be the probability that firm 1 takes the type PC_1 action. Similarly, let $\alpha_{2p} = prob[A \mid AC_2]$. Let σ_{G2p} be the probability that GOV chooses S when it receives the signal SAC_2. Let β_{1p} be the probability that firm 2 chooses S if firm 1 takes the type PC_1 action and GOV chooses $\neg S$. Similarly, let $\beta_{2p} = prob[A \mid AC_2, \sigma_{G2p} = 0]$. If the type is PC_1, firm 2 chooses S iff $((1-r)T - L_2) > 0$. If the type is AC_2, firm 2 sues iff $rT - L_2 > 0$. Given SAC_2, GOV's net expected utility from S is

(10) $\quad -D_p(1-r)x(1-\beta_{1p}) + (1-D_p)rx(1-\beta_{2p}) - L_{GOV}$,

where $D = \frac{\alpha_{1p}p(1-q)}{\alpha_{1p}p(1-q)+\alpha_{2p}(1-p)q}$. GOV chooses S if (10) is positive, $\neg S$ if (10) is negative, and randomizes if (10) equals zero. Consider next firm 1's decision. Assume that firm 1 needs to incur L_1^S in the follow-up suit. When the type is PC_1, given β_{1p}, β_{2p}, and σ_{G2p}, firm 1 randomizes between A and $\neg A$ when

(11) $\quad \left(q + (1-q)(1-\sigma_{G2p})\right)\left[\beta_{1p}(rT - L_1) + (1-\beta_{1p})T\right] + (1-q)\sigma_{G2p}\left[rT - L_1 - (1-r)(\phi T + L_1^S)\right] = 0$.

Similarly, when the type is AC_2, firm 1 randomizes between A and $\neg A$ when

(12) $\quad \left(1 - q + q(1-\sigma_{G2})\right)\left[\beta_2((1-r)T - L_1) + (1-\beta_2)T\right] + q\sigma_{G2}\left[(1-r)T - L_1 - r(\phi T + L_1^S)\right] = 0$.

Note that firm 2's problem in the subgame does not change after introducing follow-up suits. Firm 2 always sues when the action is type AC_2, $\beta_{2p} = 1$, if $T(1-r) < L_2 < Tr$ or $L_2 < T(1-r) < Tr$, and knowing this, GOV always chooses $\sigma^*_{G2p} = 0$. If $T(1-r) < Tr < L_2$, $\beta_{1p} = \beta_{2p} = 0$. Knowing this, GOV chooses S with a probability $\sigma^*_{G2p} \in [0,1]$. As long as there are independent private suits, there are no public suits. Then, follow-up suits have no effect since they only occur after public suits. Thus, follow-up suits only have an effect in region E where there are no independent private suits.

In region E, $(1-r)T < rT < L_2 < L_1$, so $\beta_1^* = \beta_2^* = 0$, and the game reduces to the game of pure public enforcement. For a given σ_{G2p}, it is always less costly to take an action if the action is type PC_1 than if it is type AC_2. Hence, firm 1 always chooses A when the type is PC_1, and randomizes when the type is AC_2. From (11) and (12), we get that

(13) $\quad \alpha^*_{2p} = \frac{p(1-q)}{(1-p)q}\left(\frac{(1-r)x+L_{GOV}}{rx-L_{GOV}}\right)$ and $\sigma^*_{G2p} = \frac{T}{q(r(1+\phi)T+rL_1^S+L_1)} < \sigma^*_{G2} = \frac{T}{q(rT+L_1)}$

Thus, as modeled, follow-up suits only decrease the probability of public suits under public enforcement, and do not change any other qualitative result reported in the main text.

Effort by the Public Enforcer. GOV may be able to make an effort to improve court accuracy. We model GOV's effort choice in the simplest way. Once GOV decides to sue, it chooses a level of effort, y , which affects court accuracy, $r(y)$. Assume $r'(y)>0$. Effort is costly, and thus, $L_{GOV}(y)=F+C(y)$, where F is GOV's fixed cost of litigation, and $C(y)$ is its variable cost. If it sues firm 1, GOV's optimization problem is

(14) $\underset{y}{Max}\ Dr(y)x-(1-D)(1-r(y))-L_{GOV}(y)$.

Assuming that the cost function $C(y)$ is sufficiently convex, the optimal effort level $y^*>0$ satisfies $r'(y^*)x=L_{GOV}{}'(y^*)$. Let $r^*=r(y^*)$ and $L^*_{GOV}=F+C(y^*)$ be the level of accuracy and the level of the litigation cost at the optimal effort level. Then GOV chooses to sue if and only if

(15) $E(S)-E(\neg S)>0 \Leftrightarrow (r^*-D+D\beta_1(1-r)-(1-D)\beta_2 r)x-L^*_{GOV}>0$,

where r^* is court accuracy if GOV sues, and r is court accuracy if firm 2 sues. The effort choice by GOV may induce a difference between court accuracy under public and under private enforcement. We now examine how this affects the results in the paper.

It is straightforward to see that Proposition 1 is unchanged except that r and L_{GOV} are replaced by r^* and L^*_{GOV}, which are defined at the optimal y^*. Proposition 2 may or may not be affected depending on whether $(r^*-r)x-L^*_{GOV}$ ><0. In regions A, B, and D, $\beta_1^*=\beta_2^*=1$, and thus from (15), GOV sues if and only if $(r^*-r)x-L^*_{GOV}>0$. If $(r^*-r)x-L^*_{GOV}<0$, GOV does not sue, and the equilibria in regions A, B, and D are the same as they were in the model without an effort choice for GOV. However, if the improvement in court accuracy from GOV's effort, (r^*-r), is sufficiently large, GOV would choose to sue, $\sigma^*_{G2}=1$.

Thus, the SPE of the game with private and public enforcement are modified as follows:

(A) If $L_2<(1-r)T<L_1<rT$, the unique SPE is $\alpha_1^*=1$, $\alpha_2^*=0$, $\sigma^*_{G2}=1$, and $\beta_1^*=\beta_2^*=1$.

(B) If $L_2<(1-r)T<rT<L_1$, $\alpha_1^*\geq 0, \alpha_2^*=0$, $\sigma^*_{G2}=1$, and $\beta_1^*=\beta_2^*=1$.

(C) If $(1-r)T<L_2<rT<L_1$ or $(1-r)T<L_2<L_1<rT$, $\alpha_1^*=1$, $\alpha_2^*=0$, $\sigma^*_{G2}=0$, $\beta_1^*=0$, $\beta_2^*=1$.

(D) If $L_2<L_1<(1-r)T<rT$, $\alpha_1^*=1,\alpha_2^*\leq 1$, $\sigma^*_{G2}=1$, and $\beta_1^*=\beta_2^*=1$.

(E) If $(1-r)T<rT<L_2<L_1$, $\beta_1^*=\beta_2^*=0$ and outcomes are as under pure public enforcement.

Notice that, if court accuracy is sufficiently improved through GOV's effort, then GOV always sues in regions A, B, and D, even though firm 2 also sues whenever firm 1 takes an anticompetitive action but GOV does not sue. With improved court accuracy, when GOV sues, the court is more likely to correctly acquit firm 1 if firm 1's action was procompetitive. Hence, GOV can effectively prevent some strategic suits by suing itself. Similarly, GOV can also effectively enhance accuracy by suing itself if firm 1's action was anticompetitive. This in turn increases the probability that firm 1 takes a procompetitive action in regions A and B and reduces the probability that firm 1 takes an anticompetitive action in region D. The equilibrium in regions C and E are not affected.

Proposition 3 on the optimal mechanism under public enforcement continues to hold as stated. Due to the private firms' incentive to free-ride on public suits, the optimal public enforcement mechanism requires that GOV cannot sue unless the fee is paid, i.e. $\sigma_{G2}^{*}=0$ (see page 16 for the explanation). Consider a fee *f* satisfying $(1-r)T<f<rT$. Given that $\sigma_{G2}^{*}=0$, in regions A, B, and D, paying the fee is not profitable for firm 2, irrespective of the type of action taken by firm 1, that is,

(16) $$E(fee)-E(\neg fee)=-\sigma_{G1}\left[(r^{*}-r)T+f-L_{2}\right]<0$$

for any $\sigma_{G1}\in(0,1]$, since $(r^{*}-r)>0$ and $L_{2}<(1-r)T<f$ in these regions. As firm 2 has no incentive to pay the fee, the fee would not work to induce truthful information revelation, and most private suits will continue to be strategic in these regions. Hence, as before, optimal public enforcement requires no private enforcement.

Without the possibility of private enforcement,

(17) $$PC:E(fee)-E(\neg fee)=-\sigma_{G1}\left[f-(1-r^{*})T\right]<0 \text{ and } AC:E(fee)-E(\neg fee)=-\sigma_{G1}\left[f-r^{*}T\right]>0$$

for any $\sigma_{G1}\in(0,1]$. Thus, firm 2 pays the fee if and only if the action was anticompetitive, so $\tilde{D}_{1}=0$ and $\tilde{D}_{2}=1$. Under these beliefs, GOV always sues only if the fee is paid since

(18) $$E(S)-E(\neg S)\big|_{fee}>0\Leftrightarrow r^{*}x-L_{GOV}^{*}+f>0 \text{ and } E(S)-E(\neg S)\big|_{\neg fee}<0\Leftrightarrow)-(1-r^{*})x-L_{GOV}^{*}<0.$$

Knowing this, firm 1 never takes a type AC_{2} action and only takes a type PC_{1} action. That is, the separating equilibrium with the fee for public lawsuit is perfectly revealing of the type, so public enforcement can achieve the optimum. In this separating equilibrium, $\lambda_{1}^{*}=0$, $\lambda_{2}^{*}=1$, $\sigma_{G1}^{*}=1$, $\sigma_{G2}^{*}=0$, $\tilde{\alpha}_{2}^{*}=0$, $\tilde{\alpha}_{1}^{*}=1$, and $\tilde{\beta}_{1}=\tilde{\beta}_{2}=0$. Thus, Proposition 3 holds exactly as it did before.

Proposition 4 also stays the same as it only depends on court accuracy under private suits, r.